A proof of the irrationality of $\pi$ and the rational powers of $e$

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Abstract

In this note we give a very short proof of irrationality of $\pi$, and a proof of the irrationality of the rational powers of $e$. The canonical proof of the irrationality of $\pi$ is due to Lambert (1767), and the one for $e$ to Euler (1737).

Keywords : Number theory, irrationality, $\pi$, $e$.

1. The irrationality of $\pi$

In this first section we give a proof of the irrationality of $\pi$ which we gave first in [1]. We begin with the following:

**Theorem 1.1.** $\pi^2$ is irrational.

**Proof.** We suppose that $\pi^2$ is rational i.e. we suppose that there exist $a, b \in \mathbb{N}$, with $(a, b) = 1$, such that $\pi^2 = \frac{a}{b}$, because $\pi^2$ is positive.

Then there exists a number $m \in \mathbb{N}$ such that $\pi \frac{a^m}{m!} < 1$ ($\pi$ is a finite quantity and the succession with generic term $\frac{a^n}{n!}$ is infinitesimal, for

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