

Optimal pricing and replenishment policy for a deteriorating item in a two-echelon supply chain

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Abstract

This paper deals with an emerging problem that jointly determines the optimal retail price and inventory replenishment policy in a manufacturer-retailer channel. We formulate both the centralized and the decentralized decision-making policies in the two-echeloned supply chain with an aim to maximizing the total profits. The demand in the retail end is assumed to be price-dependent and the item is subject to continuously exponential decay. We derive the optimal solutions, prove their optimality, and carry out numerical study. In addition, a profit-sharing mechanism, through a quantity discount scheme, is proposed so that Pareto improvement, i.e., one party is better-off and the other is not worse-off, can be achieved among channel participants.

Keywords : *Inventory, pricing, replenishment, channel coordination, deterioration.*

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1. Introduction

The ultimate goal of channel coordination mechanism is to optimize the entire system and create a win-win relationship among the partners in a supply chain. In order to achieve effective supply chain cooperation and quick response to the customers' demand, most enterprises emphasize on enhancing business information sharing, simplifying core processes, and streamlining cross company operations. However, a supply chain involves several members with a self-interest while channel-conflict objective. One of the most challenging works is to determine the optimal production, replenishment, and pricing policy at a time, that are acceptable to both the upstream and downstream entities in the supply chain. In what follows, we review the channel coordination models that are most closely related to our study and classified them into three sub-categories: (1) vendor-buyer coordination, (2) deteriorating inventory management, and (3) pricing and lot-sizing coordination.

The vendor-buyer problem focuses on channel coordination issues for inventory replenishments between the upstream and downstream entities in the supply chain, with the objective of minimizing the total channel-wide costs. Some of the representative work in this stream includes Banerjee [3], Goyal [30], and Banerjee and Kim [2] who dealt with the problem under a single-vendor and single-buyer setting, and Banerjee and Banerjee [1], Lu [15], Viswanathan and Piplani [28], and Das and Goyal [8] who undertook the problem with single-vendor and multiple-buyer distribution networks. More recently, Mishra [7] generalized the model proposed by Viswanathan and Piplani [28] to allow for a selective discount policy that can reduce the supplier's costs. Khouja [17] extended past research by considering an integrated three-staged supply chain with multi-vendors and multi-buyers. A comprehensive review for this stream of research is given in Goyal and Gupta [29]. However, the aforementioned literature does not take the deteriorating effect of on-hand inventory level into account.

Deterioration is a fact of life in inventory items, such as volatile liquids, agricultural products, radioactive substances, films, drugs, blood, fashion goods, electronic components, and high-tech products. These items are subject to depletion by phenomena other than demand; i.e. through spoilage, shrinkage, decay, and obsolescence. Ghare and Schrader [21] extended the classical economic order quantity (EOQ) model, by considering the combined effects of demand, usage, and

linear decay. Covert and Philip [24] used the variable deterioration rate of the two-parameter Weibull distribution, to formulate their model, under the assumptions of a constant demand rate with no shortages allowed. Tadikamalla [22] employed gamma distributed deterioration rate under constant demand over time without shortages. Moon and Lee [11] proposed an economic order quantity model with a normally distributed deterioration rate. Hariga [16], Jalan et al. [6], Teng et al. [13], and Wu [14] also extended some EOQ-based deteriorating inventory models by considering a time-varying demand function, with or without shortages. Dave and Patel [31] proposed a variant of the EOQ model, under time-proportional demand, with no shortages allowed; their objective was to determine the optimal replenishment cycle length and time-varying lot sizes over a finite planning horizon. Sachan [25] extended their model by allowing shortages, and Bahari-Kashani [9] generalized the problem by permitting variations in both replenishment cycle length and order lot size. The reviewed literature considers only the replenishment problem for a single entity (manufacturer or retailer); while we consider the coordinated pricing and replenishment problem in a two-echelon context.

The problem of coordinating pricing and inventory replenishment has received a great deal of attentions in recent years. Cohen [18] developed a joint replenishment and pricing model for a retailer who sells a perishable item in the marketplace with the deterministic demand. Under a monopolistic environment, Rajan et al. [4] developed the optimal replenishment cycle and retail price of a deterioration item for the retailer under the assumption which the selling price of the product was allowed to vary over the cycle. Abad [20] extended their model by including the strategy of partial backlogged, and Wee [10] developed a more realistic Weibull deteriorating inventory model with quantity discount, pricing, and partial backordering characters. More recently, Jørgensen and Kort [26] have dealt with an optimal pricing and replenishment problem with the key assumption which the in-store stock of the product could affect a retailer's demand. Papachristos and Skouri [27] generalized the work of Wee [10] by considering the convex decreasing demand and a variable backloging rate. In this paper, we have extended these works by considering an exponential decay product with price-dependent demand in a two-echelon supply chain. Our objective is to determine the optimal price and the replenishment cycle of the item, and investigate the effects on profit improvements in the chain under the centralized and the decentralized decision policies.

The organization of this paper is as follows. Section 2 outlines the problem and summarizes the necessary assumptions and notations. The mathematical models for both the decentralized and the centralized policies are developed in section 3. A discussion of the optimal properties and a search procedure under a linear demand assumption is given in section 4. A numerical study is given in section 5 which provides qualitative insight into the structure of the solutions. A quantity discount scheme is also provided in this section, which serves as a profit-sharing mechanism for the participants in the channel. Conclusions are given in section 6.

2. Assumptions and notations

This paper considers a two-echelon supply chain consisting of a manufacturer and a retailer who sells a deteriorating item in the marketplace. The demand rate of the item is assumed to be a function of the retail price and the item is subject to exponential decay. The retailer places the replenishment orders with the exclusive manufacturer based on an EOQ policy. The manufacturer purchases the raw material, which is also subject to exponential decay, from the outside vendors, and uses a lot-for-lot production policy to fulfill retailer's demand. In this particular setting, the length of the manufacturer's production cycle was equal to the length of the retailer's replenishment cycle. A reasonable assumption in the two-stage capacity-constrained production/retailing system was to allow the production rate to be greater than or equal to the demand rate in the retailer, and there was no in-transit inventory between the upstream and downstream entities in the supply chain. The logistic and cash flows of this manufacturer-retailer supply chain are illustrated in Figure 1.

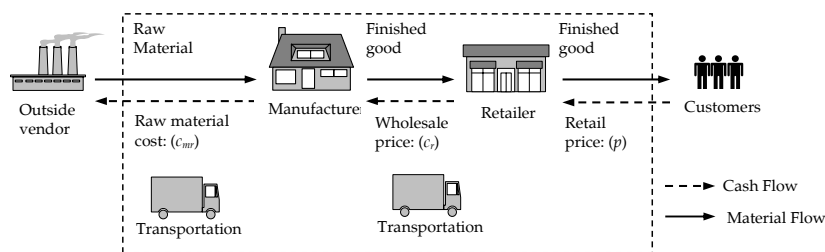


Figure 1

The logistic and cash flows in the manufacturer-retailer supply chain

The purpose of this study is to determine the optimal retail price of the finished item for end customers, and the replenishment (production) cycle for the retailer (the manufacturer) with an aim at maximizing the total profit of the channel. Before presenting the mathematical models, we define the notations used throughout the paper.

- T = The retailer's replenishment cycle of the finished item, i.e., the manufacturer's production cycle, which is a decision variable;
 t_m = The manufacturer's starting production time for the finished item;
 S_r = The setup cost per order for the retailer;
 S_m = The setup cost per lot for the manufacturer;
 s_{mr} = The ordering cost of raw material for the finished item per lot for the manufacturer;
 p = The retail price of the finished item charged by the retailer, which is a decision variable;
 c_r = The purchase cost of the finished item per unit for the retailer, i.e., the selling price charged by the manufacturer;
 c_{mr} = The purchase cost of raw material for the finished item per unit for the manufacturer, i.e., the selling price charged by the outside vendor;
 h_r = The inventory holding cost of the finished item for the retailer;
 h_m = The inventory holding cost of the finished item for the manufacturer;
 h_{mr} = The inventory holding cost of the raw material for the finished item for the manufacturer;
 $D(p)$ = The demand rate of the finished item in the marketplace, which is a function of the price p ;
 ρ = The production rate of the finished item produced by the manufacturer;
 u = The usage rate of raw material for the finished item produced by the manufacturer;
 θ = The deteriorating rate of the finished item;
 θ_m = The deteriorating rate of raw material for the finished item;
 $I_r(t)$ = Inventory level of the finished item at time t for the retailer;
 $I_m(t)$ = Inventory level of the finished item at time t for the manufacturer;
 $I_{mr}(t)$ = Inventory level of raw material for the finished item at time t for the manufacturer;

- π_r = Total profit of the retailer per unit time;
 π_m = Total profit of the manufacturer per unit time;
 π_d = Total profit of the supply chain per unit time under the decentralized policy;
 π_c = Total profit of the supply chain per unit time under the centralized policy.

3. The model

In this section, the profit models of the two-echelon supply chain derived by the decentralized and the centralized policies will be presented. Under the decentralized decision-making policy, each entity within the supply chain focuses on maximizing its own profits, without considering the reaction of its counterpart and the cost that may incur. In contrast, the centralized policy simultaneously determines the retail price p and the replenishment cycle T , by considering the total profit incurred by the retailer and the manufacturer, so that the system profit is maximized. We first present the decentralized policy and then the centralized policy for the problem.

3.1 The decentralized policy

In the decentralized policy, the retailer makes the replenishment decision for the finished item based on an EOQ policy. Under this scenario, the costs associated with the retailer include purchase cost, inventory holding cost, and setup cost. The retailer's profit was generated from the revenue minus the aforementioned costs. Under the decentralized policy, the objective of the retailer is to maximize its profit by choosing the optimal replenishment cycle and retail price. The inventory behavior of the item is due to the combined effects of demand and deterioration in the replenishment cycle, and can be described by the following differential equation:

$$\frac{dI_r(t)}{dt} = -I_r(t)\theta - D(p), \quad \text{for } 0 \leq t \leq T. \quad (1)$$

Using the method proposed by Spiegel [19] and the boundary condition $I_r(T) = 0$, we can get the inventory level of the finished item at time t :

$$I_r(t) = \frac{D(p)}{\theta} (e^{\theta(T-t)} - 1), \quad \text{for } 0 \leq t \leq T. \quad (2)$$

In each replenishment cycle, the purchase cost and inventory holding

cost of the finished item can be expressed by Eqs. (3) and (4), respectively, as follows:

$$c_r I_r(0) = \frac{c_r D(p)}{\theta} (e^{\theta T} - 1), \quad (3)$$

and

$$h_r \int_0^T I_r(t) dt = \frac{h_r D(p)}{\theta} \left[\frac{1}{\theta} (e^{\theta T} - 1) - T \right]. \quad (4)$$

The term $I_r(0)$ in Eq. (3) denotes the inventory level at the beginning of the replenishment cycle for the retailer.

The profit per unit time of the retailer can be presented with the revenue minus the setup, purchase and inventory holding costs over the replenishment cycle, and all divided by the cycle length. Neglecting third and higher order terms of θT in the Taylor series expansion of $e^{\theta T}$, the retailer's unit profit function can be expressed below:

$$\pi_r = (p - c_r)D(p) - \frac{S_r}{T} - \frac{(h_r + c_r\theta)D(p)T}{2}. \quad (5)$$

Since profit model (5) is a multivariate function, and it has the optimal solutions T^* and p^* which can maximize the retailer's profit when the concave properties and Hessian matrix condition (Khuri [5]) are satisfied. We show the optimal replenishment cycle T^* and retail price p^* forthwith and demonstrate the optimal properties in the following section. For a fixed price, the corresponding optimal replenishment cycle of each item can be obtained by differentiating Eq. (5) with respect to T , and setting the result equal to zero:

$$T^* = \left(\frac{2S_r}{(h_r + c_r\theta)D(p)} \right)^{1/2}. \quad (6)$$

The multivariate problem of maximizing the profit function π_{dr} can be reduced to a univariate problem by substituting the value T^* into Eq. (5):

$$\pi_r = (p - c_r)D(p) - [2S_r(h_r + c_r\theta)D(p)]^{1/2}. \quad (7)$$

Then, the optimal retail price of each item can be obtained by setting the first derivative of equation (7) equal to zero:

$$p^* = \left(\frac{S_r(h_r + c_r\theta)}{2D(p)} \right)^{1/2} + c_r - \frac{D(p)}{dD(p)/dp}. \quad (8)$$

While receiving the retailer's order, the manufacturer adopts a lot-for-

lot production policy to fulfill the retailer's demand. For each production run, it results the inventory holding cost and setup costs for the finished item. Furthermore, the manufacturer requires additional purchase, holding and ordering costs for the raw materials required to produce the finished good. During the production cycle, the change of inventory levels of each raw material and finished item of the manufacturer are influenced by the combined effects of production and deterioration, and their instantaneous states can be described below:

$$\frac{dI_{mr}(t)}{dt} = -u\rho - I_{mr}(t)\theta_m, \quad \text{for } t_m \leq t \leq T, \quad (9)$$

and

$$\frac{dI_m(t)}{dt} = \rho - I_m(t)\theta, \quad \text{for } t_m \leq t \leq T, \quad (10)$$

where t_m and T are the starting and the stopping production times, respectively. Derivation of the starting time t_m is detailed in Chen and Chen [12] which is

$$t_m = T - T \left(\frac{D(p)}{\rho} \right)^{1/2}. \quad (11)$$

The inventory level of each raw material and finished item at time t can be obtained by the boundary conditions $I_{mr}(T) = 0$, $I_m(t_m) = 0$ and the method proposed by Spiegel [19].

$$I_{mr}(t) = \frac{u\rho}{\theta_m} (e^{\theta_m(T-t)} - 1), \quad \text{for } t_m \leq t \leq T \quad (12)$$

$$I_m(t) = \frac{\rho}{\theta_m} (1 - e^{\theta(t_m-t)}), \quad \text{for } t_m \leq t \leq T. \quad (13)$$

The profit function per unit time of the manufacturer can be obtained by the similar procedure developed for the retailer and expressed as follows:

$$\pi_m = c_r D(p) - \frac{(S_m + s_{mr})}{T} - c_{mr} u \rho - [c_{mr} u \rho \theta_m + h_{mr} u] D(p) \frac{T}{2}. \quad (14)$$

Summing up equations (5) and (14) yields the profit model for the supply chain under the decentralized policy:

$$\pi_d = \pi_r + \pi_m. \quad (15)$$

3.2 The centralized policy

In the centralized policy, the replenishment cycle and retail price for the finished item are determined jointly by the members in the supply chain. The aim of the channel parties in this policy is to determine the optimal values of T and p of the finished item so that the channel-wide profit is maximized. The system profit per unit time of the centralized policy is

$$\begin{aligned}\pi_c &= \pi_r + \pi_m \\ &= pD(p) - \frac{(S_r + S_m + s_{mr})}{T} - c_{mr}u\rho \\ &\quad - [c_{mr}u\rho\theta_m + (h_r + c_r\theta + h_m + h_{mr}u)D(p)]\frac{T}{2}.\end{aligned}\quad (16)$$

Similarly, Eq. (16) also has the extreme values T^{**} and p^{**} which can maximize the system's profit, we present the optimal solutions of the profit model (16) forthwith and leave the derivations of the optimal properties in the following section. Under a fixed price, the length of a optimal replenishment cycle of the finished item can be obtained by differentiating Eq. (16) with respect to T , and setting the result equal to zero:

$$T^{**} = \left\{ \frac{2(S_r + S_m + s_{mr})}{c_{mr}u\rho\theta_m + (h_r + c_r\theta + h_m + h_{mr}u)D(p)} \right\}^{1/2}.\quad (17)$$

The system's unit profit can be rewritten by substituting the value of T^{**} into Eq. (17):

$$\begin{aligned}\pi_c &= pD(P) - c_{mr}u\rho - [2(S_r + S_m + s_{mr}) \\ &\quad \times (c_{mr}u\rho\theta_m + (h_r + c_r\theta + h_m + h_{mr}u)D(p))]^{1/2}.\end{aligned}\quad (18)$$

After setting the first derivative of equation (18) equal to zero, we can obtain the optimal retail price of the finished item

$$\begin{aligned}p^{**} &= (h_r + c_r\theta + h_m + h_{mr}u) \\ &\quad \times \left\{ \frac{S_r + S_m + s_{mr}}{2[c_{mr}u\rho\theta_m + (h_r + c_r\theta + h_m + h_{mr}u)D(p)]} \right\}^{1/2} \\ &\quad - \frac{D(p)}{dD(p)/dp}.\end{aligned}\quad (19)$$

4. A linear demand case

Next, we prove that the profit functions of the decentralized and

centralized policies are concave in the replenishment cycle (T) and retail price (p) under the linear demand case. We then discuss the necessary conditions to obtain the optimal solutions which can maximize the profit functions. An iterative search procedure is also proposed to solve the problems based on the concave property.

4.1 Optimal property

In this section, the concave properties of the profit functions have been investigated. Since the optimal properties of the centralized policy are similar to the decentralized policy, we only demonstrate the relevant characters of the decentralized policy and ignore these of the centralized policy.

Lemma 1. *The unit profit function of the retailer, π_r , for the decentralized policy is concave in T .*

Proof. The concave property of the retailer's unit profit function can easily be verified by substituting the linear demand function (i.e., $D(p) = a - bp$, where a and b are positive constants) into Eq. (5) and taking the second-order derivative with respect to T . It result

$$\frac{\partial^2 \pi_r}{\partial T^2} = \frac{-2S_r}{T^3}. \quad (20)$$

The result of Eq. (20) is negative since $S_r > 0$ and $T > 0$. It reveals that the retailer's unit profit function is concave in T . \square

Lemma 2. *The unit profit function of the retailer, π_r , for the decentralized policy is concave in p .*

Proof. Substitute the linear demand function into Eq. (5) and take the second-order derivative with respect to p , we can obtain

$$\frac{\partial^2 \pi_r}{\partial p^2} = -2b. \quad (21)$$

Since the result of Eq. (21) is negative, thus the retailer's unit profit function is concave in p . \square

Proposition 1. *Equation (5) has the optimal values T^* and p^* which can maximize the retailer's unit profit when the concave properties and Hessian matrix condition $(\partial^2 \pi_r / \partial T^2)(\partial^2 \pi_r / \partial p^2) - (\partial^2 \pi_r / \partial T_p)^2 > 0$ are satisfied.*

Proof. The concave properties of Eq. (5) have been obtained by Lemmas 1 and 2. After some mathematical operations, the Hessian matrix condition can be reduced as below:

$$T < \sqrt[3]{\frac{16S_r}{b(h_r + c_r\theta)^2}}. \quad (22)$$

Based on the argument above, the profit model (5) has the optimal solutions T^* and p^* under the condition of T , Eq. (22) is hold. \square

4.2 Search procedure

In what follows, we summarized an optimal solution search procedure to obtain the optimal retail price p^* and its corresponding replenishment cycle (T^* or T^{**}). We assume that the demand rate in the marketplace decreases in the retail price, i.e., $D(p) = a - bp$. Under this assumption, the polynomial equation of the retail price of the finished item under decentralized policy can be obtained by substituting the value of $D(p)$ into Eq. (8) and simplifying into:

$$p^3 - \left(\frac{2a}{b} + c_r\right)p^2 + \left(\frac{a}{b} + c_r\right)(5a + bc_r)\frac{p}{4b} + \frac{S_r(h_r + c_r\theta)}{8b} - \frac{a}{4b}\left(\frac{a}{b} + c_r\right)^2 = 0. \quad (23)$$

Because Eq. (23) is a cubic equation of p , we only consider the rational roots of its solutions when the search procedure executes. The search procedure of the centralized policy follows a similar pattern of the decentralized policy and has, therefore, been omitted. The step by step process for the search procedure is outlined below.

- Step 0.* Calculate the solutions of the retail price by solving Eq. (23). Then, substitute each retail price's solution into Eq. (6) and solve it to obtain the corresponding replenishment cycle T .
- Step 1.* Substitute each retail price and its corresponding replenishment cycle obtained from *Step 0* into the profit model Eq. (15).
- Step 2.* Compare all total unit profits obtained in Step 1. After making this comparison, the maximum total unit profit was obtained; the corresponding optimal retail price and replenishment cycle are represented by p^* and T^{**} .
- Step 3.* Report the optimal solutions: the optimal retail price p^* , the optimal replenishment cycle T^* , and terminate the search procedure.

5. Numerical study

In this section, a numerical example is presented to illustrate the mathematical behavior of the proposed models and gain some insights into the problem being studied. Consider a retailer requiring an item from a manufacturer, with the linear demand $D = a - bp$, where $a = 450$ and $b = 3.5$, respectively. The corresponding production and deteriorating rates for the item per month are 350 and 0.18. The setup cost for each order placed by the retailer is \$500, and the unit purchase and monthly holding costs are \$45 and \$2.4, respectively. As for the cost components facing the manufacturer, the setup cost for each production run and monthly holding costs for the finished item are \$800 and \$1.1. The raw material's ordering, purchase and monthly holding costs are \$25, \$16 and \$0.5, and the usage and deteriorating rates are 0.15 and 0.05, respectively.

In order to investigate the profit growth efforts on the retailer, the manufacturer and the channel, the results obtained from the decentralized policy are used as the base line to compute the improvements generated by the centralized policy. Table 1 reports the numerical results including the optimal replenishment cycle, retail price, and the profit generated by both policies. It reveals the manufacturer and the system are better-off with 84.93% and 19.35% profit improvements, respectively, while the retailer suffering a 38.65% decrease in profit by adopting the centralized policy. It is obvious that the retailer would not accept such policy, although channel-wide performance is better-off. Among the possible solutions implemented in practice, quantity discounts offered by the manufacturer to the retailer seems to be a promising benefit sharing arrangement (see Lal and Staelin [23]).

Table 1
The profit growth effects on the retailer, the manufacturer and the channel

Decentralized policy	Centralized policy	Profit improvement
$T^* = 0.8286$	$T^{**} = 1.0126$	
$P^* = 88.9$	$P^{**} = 66.4$	
Profit of retailer = 4887	Profit of retailer = 2998	(-38.65%)
Profit of manufacturer = 4322	Profit of manufacturer = 7993	(+84.93%)
Profit of system = 9209	Profit of system = 10991	(+19.35%)

The second experiment was conducted to examine whether a properly designed quantity discount scheme can be an effective mechanism to achieve Pareto improvements among channel members, i.e., one party is better off and the other is not worse off. The quantity discount schedule, outlined in Table 2, is adopted from Lal and Staelin [23], which was implemented herein to serve as the profit-sharing mechanism for the centralized policy. Table 3 reports the numerical results after quantity discount. Under the new arrangements, the percentages of profit improvements for the retailer, the manufacturer, and the channel are 15.02, 67.95, and 19.35, respectively. This benefit sharing arrangement makes the proposed policy (i.e., the centralized policy) acceptable by both channel members.

Table 2

Channel rebate schedule offered by the manufacturer to the retailer

Original price	Discount price	Break point
45	38	75

Table 3

Profit sharing results after quantity discount

	Decentralized policy	Centralized policy	After quantity discount
Replenishment cycle	$T^* = 0.8286$	$T^{**} = 1.0126$	$T^{**} = 1.0126$
Retailer price	$P^* = 88.9$	$P^{**} = 66.4$	$P^{**} = 66.4$
Profit of retailer	4887	2998 (-38.65%)	5621 (+15.02%)
Profit of manufacturer	4322	7993 (+84.93%)	7259 (+67.95%)
Profit of system	9209	10991 (+19.35%)	10991 (+19.35%)

6. Conclusion

This study has developed two decision-making models that determine the optimal pricing, inventory replenishment, and production policies in a supply chain. These policies offer structural and quantitative insight into the problem of integrating pricing, vendor-buyer coordination, and deteriorating inventory management. The numerical example presented carries some implications for management. A centralized decision policy was always found to be superior to the decentralized decision policy in terms of profit improvement. In addition, a profit-sharing mechanism through a quantity discount scheme has been proposed to

offset the increased costs to the retailer so that Pareto improvements can be achieved.

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