A decision-making procedure on process centering: a lower bound of the estimated accuracy index

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Abstract

Process accuracy index C_a has been proposed in the manufacturing industry to provide numerical measures on assessing process performance with respect to the accuracy. Investigations on process centering (the ability to cluster around the specification mid-point) in existing engineering statistics and quality assurance literature reveal conservatively rare. In this paper, a natural estimator of the accuracy index is considered under the normality assumption. The exact probability density function and the r-th moment raised from a non-central chi-squared distribution of the estimated index are derived. A decision-making procedure on process centering for in-plant applications is constructed. Values in lower bound of the estimated index required to ensure the process reaching a desirable accuracy level of the time are also tabulated. A practical example is presented to illustrate how the proposed reliable procedure may be applied.

Keywords : *Accuracy index, decision, performance, process centering.*

1. Introduction

Process capability indices C_p, k, and C_{pk} are three basic indices (Kane, 1986) that have been widely used in the manufacturing industry to provide numerical measures on process potential and performance. While C_p measures the overall process variation relative to the manufacturing

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tolerance, k measures the degree of process departure from the target value. The three basic indices C_p, k, and C_{pk} are defined as:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma}$$

$$k = \frac{|\mu - m|}{d}$$

$$C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\}, \tag{1}$$

where USL and LSL are the upper and the lower specification limits, μ is the process mean, σ is the process standard deviation, m is the mid-point between the upper and the lower specification limits, and d is half length of the specification interval $[\text{LSL}, \text{USL}]$. That is, $m = (\text{USL} + \text{LSL})/2$ and $d = (\text{USL} - \text{LSL})/2$.

Pearn et al. (1998) considered a transformation of k defined as $C_a = 1 - k$ which measures the degree of process centering (the ability to cluster around the center), which has been regarded to the process accuracy index. Table 1 displays various C_a values and the corresponding ranges of the departure magnitude of μ. For example, $C_a = 1$ indicates that the process is perfectly centered $(\mu = m)$, $C_a > 1/2$ indicates that μ is within half of the specification interval, and $C_a = 0$ indicates that μ is on the specification limits $(\mu = \text{USL}, \text{ or } \mu = \text{USL})$. If $C_a < 0$, then it indicates that μ falls outside the specification interval. Obviously, the process is severely off-center in this case and it needs an immediate troubleshooting. Table 2 displays the corresponding values of C_a under various commonly used characteristic parameters.

Table 1
C_a values and the corresponding ranges of μ

C_a value	Range of μ
$C_a < 0.00$	$\mu < \text{LSL}$ or $\mu > \text{USL}$
$C_a = 0.00$	$\mu = \text{LSL}$ or $\mu = \text{USL}$
$0.00 < C_a < 0.25$	$3d/4 < \|\mu - m\| < d$
$0.25 < C_a < 0.50$	$d/2 < \|\mu - m\| < 3d/4$
$0.50 < C_a < 0.75$	$d/4 < \|\mu - m\| < d/2$
$0.75 < C_a < 1.00$	$0 < \|\mu - m\| < d/4$
$C_a = 1.00$	$\mu = m$

Table 2

The corresponding values of C_a under various characteristic parameters

d/σ	$\lvert\mu - m\rvert/\sigma$								
	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
0.5	1.0000	0.5000	0.0000	–0.500	–1.000	–1.500	–2.000	–2.500	–3.000
1.0	1.0000	0.7500	0.5000	0.2500	0.0000	–0.250	–0.500	–0.750	–1.000
1.5	1.0000	0.8333	0.6667	0.5000	0.3333	0.1667	0.0000	–0.167	–0.333
2.0	1.0000	0.8750	0.7500	0.6250	0.5000	0.3750	0.2500	0.1250	0.0000
2.5	1.0000	0.9000	0.8000	0.7000	0.6000	0.5000	0.4000	0.3000	0.2000
3.0	1.0000	0.9167	0.8333	0.7500	0.6667	0.5833	0.5000	0.4167	0.3333
3.5	1.0000	0.9286	0.8571	0.7857	0.7143	0.6429	0.5714	0.5000	0.4286
4.0	1.0000	0.9375	0.8750	0.8125	0.7500	0.6875	0.6250	0.5625	0.5000
4.5	1.0000	0.9444	0.8889	0.8333	0.7778	0.7222	0.6667	0.6111	0.5556
5.0	1.0000	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000
5.5	1.0000	0.9546	0.9091	0.8636	0.8182	0.7727	0.7273	0.6818	0.6364
6.0	1.0000	0.9583	0.9167	0.8750	0.8333	0.7917	0.7500	0.7083	0.6667

2. The sampling distribution of the estimated accuracy index

2.1 *The sampling distribution derived from a folded nonnal distribution*

Pearn et al. (1998) considered the natural estimator $\hat{C}_a$ defined as

$$\hat{C}_a = 1 - \frac{|\bar{X} - m|}{d}, \tag{2}$$

where $\bar{X} = \left(\sum_{i=1}^{n} X_i\right)/n$ is the sample mean, a conventional estimator of the process mean μ. The estimator can be alternatively written as

$$\hat{C}_a = 1 - \frac{1}{2\sqrt{n}C_p}\left(\frac{\sqrt{n}|\bar{X} - m|}{\sigma}\right). \tag{3}$$

We note that the statistic $(\sqrt{n}|\bar{X} - m|/\sigma)$ follows a folded normal distribution if the characteristic measurements $\{X_1, X_2, \ldots, X_n\}$ were chosen randomly from $N(\mu, \sigma^2)$, a normal distribution with mean μ and variance σ^2. The probability density function (pdf) of $\hat{C}_a$ can be expressed as (Pearn et al., 1998):

$$f(x) = 6C_p\sqrt{\frac{n}{2\pi}}\cosh\left(\frac{(1-x)\delta}{1-C_a}\right)\exp\left\{-\frac{\delta}{2}\left(1 + \frac{(1-x)^2}{(1-C_a)^2}\right)\right\}, \tag{4}$$

where $-\infty < x \leqq 1$ and $\delta = n[(\mu - m)/\sigma]^2 = 9n[C_p(1 - C_a)]^2$. Pearn et al. (1998) also derived the first two moments and the mean-squared error of $\hat{C}_a$. as presented in the following:

$$E(\hat{C}_a) = C_a - \frac{1}{3C_p}\sqrt{\frac{2}{n\pi}}\exp\left(-\frac{\delta}{2}\right) + 2(1 - C_a)\Phi(-\sqrt{\delta}), \tag{5}$$

$$E(\hat{C}_a^2) = C_a^2 + \frac{1}{9nC_p^2} - \frac{2}{3C_p}\sqrt{\frac{2}{n\pi}}\exp\left(-\frac{\delta}{2}\right) + 4(1 - C_a)\Phi(-\sqrt{\delta}), \tag{6}$$

$$\mathrm{MSE}(\hat{C}_a) = \frac{1}{9C_p^2} - \frac{2(1 - C_a)}{3C_p}\sqrt{\frac{2}{n\pi}}\exp - \left(\frac{\delta}{2}\right) + 4(1 - C_a)^2\Phi(-\sqrt{\delta}), \tag{7}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function (cdf) of the standard normal distribution $N(0, 1)$.

2.2 *The sampling distribution derived from a non-central chi-squared distribution*

In this section, we presented the pdf of the estimated process accuracy index in an alternative non-closed form. Assuming that the characteristic measurements $\{X_1, X_2, \ldots, X_n\}$ were chosen randomly from $N(\mu, \sigma^2)$, a normal distribution with mean μ and variance σ^2. Notice that $\bar{X} - m$ is distributed as $N(\mu - m, \sigma^2/n)$, therefore $\delta[(1 - \hat{C}_a)/(1 - C_a)]^2 = n[(\bar{X} - m)/\sigma]^2$ follows $\chi^2(1, \delta)$, a non-central chi-squared distribution with one degree of freedom and non-centrality parameter $\delta = n[(\mu - m)/\sigma]^2 = 9n[C_p(1 - Ca)]^2$. Therefore, $\hat{C}_a$ is distributed as $1 - (1 - Ca) \times \sqrt{\chi^2(1, \delta)/\delta}$. The pdf of $\hat{C}_a$ can be expressed as (see Appendix I for the derivation)

$$f(x) = \sum_{k=0}^{\infty} \left\{ \left(\frac{\sqrt{2\delta}}{(1 - C_a)\Gamma[k + (1/2)]} \right) \left[\frac{\delta}{2} \left(\frac{1 - x}{1 - C_a} \right)^2 \right]^k \times \exp\left[-\frac{\delta}{2} \left(\frac{1 - x}{1 - C_a} \right)^2 \right] \frac{(\delta/2)^k \exp(-\delta/2)}{T(k + 1)} \right\}, \quad (8)$$

where $-\infty < x \leqq 1$. Figures 1-4 show the pdfs of $\hat{C}_a$ for $n = 25$ and 50 respectively under various characteristic parameters $|\mu - m|/\sigma = 1.0, 2.0$, and $d/\sigma = 2, 4, 6$.

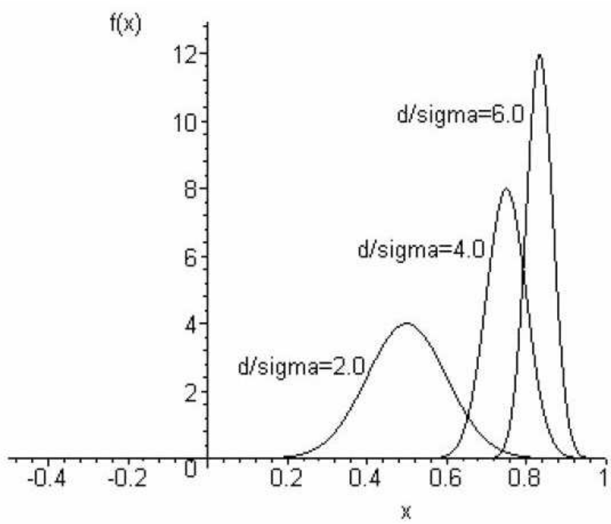

Figure 1
The pdf plot of $\hat{C}_a$ with $n = 25$, $|\mu - m|/\sigma = 1$, and $d/\sigma = 2, 4, 6$

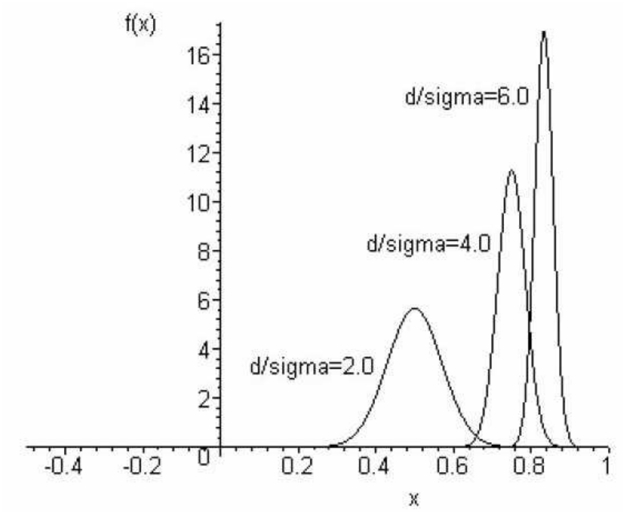

Figure 2
The pdf plot of $\hat{C}_a$ with $n = 50$, $|\mu - m|/\sigma = 1$, and $d/\sigma = 2, 4, 6$

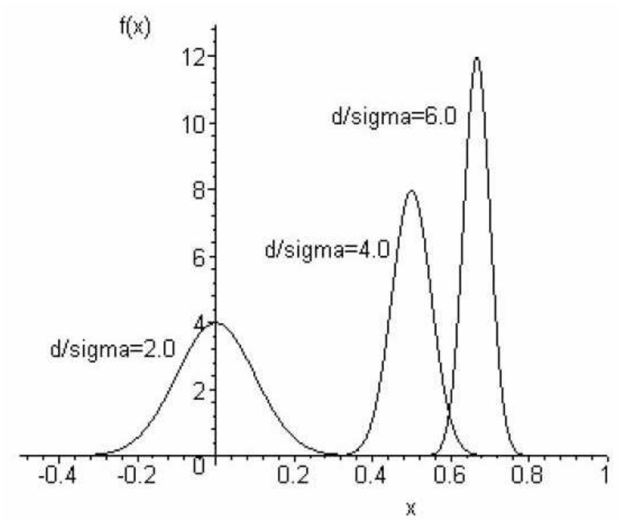

Figure 3
The pdf plot of $\hat{C}_a$ with $n = 25$, $|\mu - m|/\sigma = 2$, and $d/\sigma = 2, 4, 6$

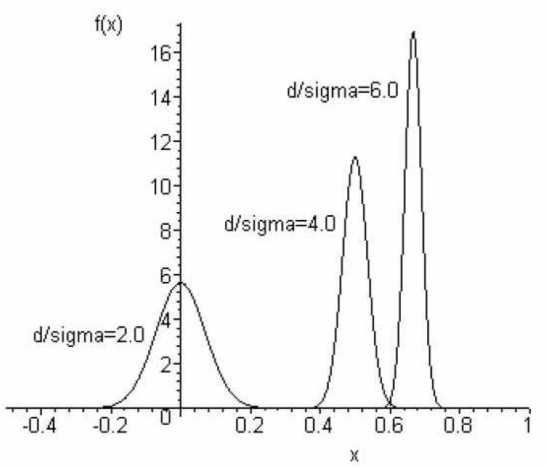

Figure 4
The pdf plot of $\hat{C}_a$ with $n = 50$, $|\mu - m|/\sigma = 2$, and $d/\sigma = 2, 4, 6$

The r-th moment of $1 - \hat{C}_a$ can be calculated as (see Appendix II for the derivation)

$$E(1-\hat{C}_a)^r = \sum_{k=0}^{\infty} \left\{ \left((1-C_a)\sqrt{\frac{2}{\delta}} \right)^2 \times \frac{T\{k+[(r+1)/2]\}}{T[K+(1/2)]} \frac{(\delta/2)^k \exp(-\delta/2)}{T(k+1)} \right\}. \quad (9)$$

Substituting $r = 1$ and $r = 2$ into expression (9) respectively, then the first two moments of $\hat{C}_a$ are given as (see appendix II for the detail derivation)

$$E(\hat{C}_a) = 1 - \sum_{k=0}^{\infty} \left\{ \frac{1}{3C_p}\sqrt{\frac{2}{n}} \frac{\Gamma(k+1)}{\Gamma[k+(1/2)]} \frac{(\delta/2)^k \exp(-\delta/2)}{\Gamma(k+1)} \right\}, \quad (10)$$

$$E(\hat{C}_a^2) = 1 + \sum_{k=0}^{\infty} \left\{ \left[\frac{2k+1}{9nC_p^2} - \frac{2}{3C_p}\sqrt{\frac{2}{n}} \frac{\Gamma(k+1)}{\Gamma[k+(1/2)]} \right] \times \frac{(\delta/2)^k \exp(-\delta/2)}{\Gamma(k+1)} \right\}. \quad (11)$$

Some numerical values of the expectation $E(\hat{C}_a)$ and the mean-squared error $\text{MSE}(\hat{C}_a) = E(\hat{C}_a - C_a)^2$ are tabulated in Tables 3(a)-3(b), respectively. It is noted that $E(\hat{C}_a) < C_a$ for all values of the characteristic parameters d/σ, $|\mu - m|/\sigma$, and sample size n. That is, $E(\hat{C}_a)$ underestimates the actual C_a values.

Table 3(a)

The expected value of $\hat{C}_a$ under various n and characteristic parameters

n	d/σ	$\|\mu-m\|/\sigma$ 0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
20	0.5	0.6432	0.4408	–0.0039	–0.5001	–1.0000	–1.5000	–2.0000	–2.5000	–3.0000
	1.0	0.8216	0.7204	0.4980	0.2500	0.0000	–0.2500	–0.5000	–0.7500	–1.0000
	1.5	0.8811	0.8136	0.6654	0.5000	0.3333	0.1667	0.0000	–1.6667	–0.3333
	2.0	0.9108	0.8602	0.7490	0.6250	0.5000	0.3750	0.2500	0.1250	0.0000
	2.5	0.9286	0.8882	0.7992	0.7000	0.6000	0.5000	0.4000	0.3000	0.2000
	3.0	0.9405	0.9068	0.8327	0.7500	0.6667	0.5833	0.5000	0.4167	0.3333
	3.5	0.9490	0.9201	0.8566	0.7857	0.7143	0.6429	0.5714	0.5000	0.4286
	4.0	0.9554	0.9301	0.8745	0.8125	0.7500	0.6875	0.6250	0.5625	0.5000
	4.5	0.9604	0.9379	0.8885	0.8333	0.7778	0.7222	0.6667	0.6111	0.5556
	5.0	0.9643	0.9441	0.8996	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000
	5.5	0.9676	0.9492	0.9087	0.8636	0.8182	0.7727	0.7273	0.6818	0.6364
	6.0	0.9703	0.9534	0.9163	0.8750	0.8333	0.7917	0.7500	0.7083	0.6667
30	0.5	0.7087	0.4714	–0.0007	–0.5000	–1.0000	–1.5000	–2.0000	–2.5000	–3.0000
	1.0	0.8543	0.7357	0.4997	0.2500	0.0000	–0.2500	–0.5000	–0.7500	–1.0000
	1.5	0.9029	0.8238	0.6665	0.5000	0.3333	0.1667	0.0000	–1.6667	–0.3333
	2.0	0.9272	0.8678	0.7498	0.6250	0.5000	0.3750	0.2500	0.1250	0.0000
	2.5	0.9417	0.8943	0.7999	0.7000	0.6000	0.5000	0.4000	0.3000	0.2000
	3.0	0.9514	0.9119	0.8332	0.7500	0.6667	0.5833	0.5000	0.4167	0.3333
	3.5	0.9584	0.9245	0.8570	0.7857	0.7143	0.6429	0.5714	0.5000	0.4286
	4.0	0.9636	0.9339	0.8749	0.8125	0.7500	0.6875	0.6250	0.5625	0.5000
	4.5	0.9676	0.9413	0.8888	0.8333	0.7778	0.7222	0.6667	0.6111	0.5556
	5.0	0.9709	0.9471	0.8999	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000
	5.5	0.9735	0.9519	0.9090	0.8636	0.8182	0.7727	0.7273	0.6818	0.6364
	6.0	0.9757	0.9560	0.9166	0.8750	0.8333	0.7917	0.7500	0.7083	0.6667

(*Contd. Table* 3(a))

n	d/σ	$\lvert\mu - m\rvert/\sigma$								
		0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
40	0.5	0.7477	0.4846	–0.0001	–0.5000	–1.0000	–1.5000	–2.0000	–2.5000	–3.0000
	1.0	0.8739	0.7423	0.4999	0.2500	0.0000	–0.2500	–0.5000	–0.7500	–1.0000
	1.5	0.9159	0.8282	0.6666	0.5000	0.3333	0.1667	0.0000	–1.6667	–0.3333
	2.0	0.9369	0.8712	0.7500	0.6250	0.5000	0.3750	0.2500	0.1250	0.0000
	2.5	0.9495	0.8969	0.8000	0.7000	0.6000	0.5000	0.4000	0.3000	0.2000
	3.0	0.9580	0.9141	0.8333	0.7500	0.6667	0.5833	0.5000	0.4167	0.3333
	3.5	0.9640	0.9264	0.8571	0.7857	0.7143	0.6429	0.5714	0.5000	0.4286
	4.0	0.9685	0.9356	0.8750	0.8125	0.7500	0.6875	0.6250	0.5625	0.5000
	4.5	0.9720	0.9427	0.8889	0.8333	0.7778	0.7222	0.6667	0.6111	0.5556
	5.0	0.9748	0.9485	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000
	5.5	0.9771	0.9531	0.9091	0.8636	0.8182	0.7727	0.7273	0.6818	0.6364
	6.0	0.9790	0.9571	0.9167	0.8750	0.8333	0.7917	0.7500	0.7083	0.6667
50	0.5	0.7743	0.4913	0.0000	–0.5000	–1.0000	–1.5000	–2.0000	–2.5000	–3.0000
	1.0	0.8872	0.7456	0.5000	0.2500	0.0000	–0.2500	–0.5000	–0.7500	–1.0000
	1.5	0.9248	0.8304	0.6667	0.5000	0.3333	0.1667	0.0000	–1.6667	–0.3333
	2.0	0.9436	0.8728	0.7500	0.6250	0.5000	0.3750	0.2500	0.1250	0.0000
	2.5	0.9549	0.8983	0.8000	0.7000	0.6000	0.5000	0.4000	0.3000	0.2000
	3.0	0.9624	0.9152	0.8333	0.7500	0.6667	0.5833	0.5000	0.4167	0.3333
	3.5	0.9678	0.9273	0.8571	0.7857	0.7143	0.6429	0.5714	0.5000	0.4286
	4.0	0.9718	0.9364	0.8750	0.8125	0.7500	0.6875	0.6250	0.5625	0.5000
	4.5	0.9749	0.9435	0.8889	0.8333	0.7778	0.7222	0.6667	0.6111	0.5556
	5.0	0.9774	0.9491	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000
	5.5	0.9795	0.9537	0.9091	0.8636	0.8182	0.7727	0.7273	0.6818	0.6364
	6.0	0.9812	0.9576	0.9167	0.8750	0.8333	0.7917	0.7500	0.7083	0.6667

Table 3(b)

The mean-squared error of $\hat{C}_a$ under various n and characteristic parameters

n	d/σ	$\lvert\mu - m\rvert/\sigma$								
		0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
20	0.5	0.2000	0.1408	0.1921	0.1997	0.2000	0.2000	0.2000	0.2000	0.2000
	1.0	0.5000	0.0352	0.0480	0.0499	0.0500	0.0500	0.0500	0.0500	0.0500
	1.5	0.0222	0.0156	0.0213	0.0222	0.0222	0.0222	0.0222	0.0222	0.0222
	2.0	0.0125	0.0088	0.0120	0.0125	0.0125	0.0125	0.0125	0.0125	0.0125
	2.5	0.0080	0.0056	0.0077	0.0080	0.0080	0.0080	0.0080	0.0080	0.0080
	3.0	0.0056	0.0039	0.0053	0.0055	0.0056	0.0056	0.0056	0.0056	0.0056
	3.5	0.0041	0.0029	0.0039	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041
	4.0	0.0031	0.0022	0.0030	0.0031	0.0031	0.0031	0.0031	0.0031	0.0031
	4.5	0.0025	0.0017	0.0024	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025
	5.0	0.0020	0.0014	0.0019	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
	5.5	0.0017	0.0012	0.0016	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017
	6.0	0.0014	0.0010	0.0013	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014
30	0.5	0.1333	0.1047	0.1320	0.1333	0.1333	0.1333	0.1333	0.1333	0.1333
	1.0	0.0333	0.0262	0.0330	0.0333	0.0333	0.0333	0.0333	0.0333	0.0333
	1.5	0.0148	0.0116	0.0147	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148
	2.0	0.0083	0.0065	0.0082	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083
	2.5	0.0053	0.0042	0.0053	0.0053	0.0053	0.0053	0.0053	0.0053	0.0053
	3.0	0.0037	0.0029	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037
	3.5	0.0027	0.0021	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	4.0	0.0021	0.0016	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021
	4.5	0.0016	0.0013	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016
	5.0	0.0013	0.0010	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013
	5.5	0.0011	0.0009	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011
	6.0	0.0009	0.0007	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009

(*Contd. Table* 3(b))

n	d/σ	$\lvert\mu - m\rvert/\sigma$								
		0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
40	0.5	0.1000	0.0846	0.0997	0.0100	0.1000	0.1000	0.1000	0.1000	0.1000
	1.0	0.0250	0.0212	0.0249	0.0250	0.0250	0.0250	0.0250	0.0250	0.0250
	1.5	0.0111	0.0094	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111
	2.0	0.0062	0.0053	0.0062	0.0062	0.0063	0.0063	0.0063	0.0063	0.0063
	2.5	0.0040	0.0034	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040
	3.0	0.0028	0.0024	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028
	3.5	0.0020	0.0017	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
	4.0	0.0016	0.0013	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016
	4.5	0.0012	0.0010	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012
	5.0	0.0010	0.0008	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
	5.5	0.0008	0.0007	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008
	6.0	0.0007	0.0006	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
50	0.5	0.0800	0.0712	0.0799	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800
	1.0	0.0200	0.0178	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
	1.5	0.0089	0.0079	0.0089	0.0089	0.0089	0.0089	0.0089	0.0089	0.0089
	2.0	0.0050	0.0044	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050
	2.5	0.0032	0.0028	0.0032	0.0032	0.0032	0.0032	0.0032	0.0032	0.0032
	3.0	0.0022	0.0020	0.0022	0.0022	0.0022	0.0022	0.0022	0.0022	0.0022
	3.5	0.0016	0.0015	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016
	4.0	0.0013	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	0.0013	0.0013
	4.5	0.0010	0.0009	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
	5.0	0.0080	0.0007	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008
	5.5	0.0001	0.0006	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
	6.0	0.0001	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006

3. A lower bound on the estimated accuracy index

To judge whether the process meets the accuracy requirement and runs under the desired quality condition, we can consider the following approach to obtain a lower bound on the estimated accuracy index. Under the normality assumption, $\delta[(1-\hat{C}_a)/(1-C_a)]^2 = n[(\bar{X}-m)/\sigma]^2$ follows $\chi^2(1,\delta)$, a non-central chi-squared distribution with one degree of freedom and non-centrality parameter $\delta = n[(\mu-m)/\sigma]^2 = 9n[C_p(1-C_a)]^2$. Denote the $(1-\alpha)\times 100$th percentile of $\chi^2(1,\delta)$ as $\chi^2_{1-\alpha}(1,\delta)$. Let $C = C(X_1, X_2, \ldots, X_n)$ be a statistic satisfies $P\{\hat{C}_a \geqq C\} = 1-\alpha$, where the confidence coefficient $1-\alpha$ does not depend on C_a. Then, C is a $100(1-\alpha)\%$ lower confidence bound for C_a. Notice that

$$\begin{aligned} P\{C_a \geqq C\} &= P\{1 - C_a \leqq 1 - C\} \\ &= P\left\{\delta\left(\frac{1-\hat{C}_a}{1-C_a}\right)^2 \geq \delta\left(\frac{1-\hat{C}_a}{1-C}\right)^2\right\} \\ &= P\left\{\chi^2(1,\delta) \geq \delta\left(\frac{1-\hat{C}_a}{1-C}\right)\right\} \\ &= 1-\alpha\,. \end{aligned} \tag{12}$$

Therefore, $\delta[(1-\hat{C}_a)/(1-C)]^2 = \chi^2_\alpha(1,\delta)$. A $100(1-\alpha)\%$ lower confidence bound of C_a can be expressed as

$$C = 1 - (1-\hat{C}_a)\sqrt{\frac{\delta}{\chi^2_\alpha(1,\delta)}}\,. \tag{13}$$

Suppose that a process is capable if $\hat{C}_a \geqq 1-(1-C_0)[\chi^2_\alpha(1,\delta)/\delta]^{1/2}$, where C_0 is the minimum value for which the underlying process is considered capable (i.e. $C_a \geqq C_0$), and we claim that the process is capable with at least $100(1-\alpha)\%$ confidence of the time. Therefore, the factor $1-(1-C_0)[\chi^2_\alpha(1,\delta)/\delta]^{1/2}$ is the recommended minimum value of the estimated accuracy index C_a in order that the process is considered accurate with at least $100(1-\alpha)\%$ confidence of the time. Tables 4(a)-4(c) present the recommended minimum values of $\hat{C}_a$ for processes to be considered capable for commonly used accuracy requirement $C_a = C_0 = 0.25, 0.50, 0.75$ under $|\mu - m|/\sigma = 0.25(0.25)1.25$ with $n = 10(10)100$ and confidence levels $1-\alpha = 0.900, 0.950, 0.975, 0.990$. A SAS program for calculating the recommended minimum value is also provided in Appendix III.

Table 4(a)
Recommended minimum values of $\hat{C}_a$ under accuracy requirement $C_0 = 0.25$ and various n, $1-\alpha$, $|\mu - m|/\sigma$ values for which the process is considered capable $(C_a \geqq C_0)$

n	$1-\alpha$	$\|\mu - m\|/\sigma$				
		0.25	0.50	0.75	1.00	1.25
10	0.900	0.98982	0.95024	0.80581	0.55395	0.20806
	0.950	0.99492	0.97436	0.86909	0.64011	0.31576
	0.975	0.99746	0.98707	0.91979	0.71482	0.40918
	0.990	0.99898	0.99481	0.96401	0.80146	0.51780
20	0.900	0.99020	0.91963	0.73932	0.46492	0.09678
	0.950	0.99509	0.94867	0.78501	0.52585	0.17294
	0.975	0.99755	0.97043	0.82464	0.57870	0.23899
	0.990	0.99902	0.98739	0.87070	0.64014	0.31580
30	0.900	0.98920	0.90024	0.70974	0.42548	0.04748
	0.950	0.99454	0.92507	0.74705	0.47523	0.10966
	0.975	0.99726	0.94644	0.77941	0.51838	0.16360
	0.990	0.99890	0.96993	0.81704	0.56855	0.22631
40	0.900	0.98756	0.88849	0.69210	0.40197	0.01809
	0.950	0.99359	0.91003	0.72442	0.44506	0.07194
	0.975	0.99677	0.92871	0.75244	0.48242	0.11865
	0.990	0.99870	0.95036	0.78503	0.52587	0.17296
50	0.900	0.98559	0.88046	0.68007	0.38593	–0.00196
	0.950	0.99230	0.89973	0.70897	0.42446	0.04620
	0.975	0.99607	0.91644	0.73404	0.45789	0.08798
	0.990	0.99842	0.93587	0.76318	0.49675	0.13656
60	0.900	0.98354	0.87454	0.67119	0.37409	–0.01677
	0.950	0.99072	0.89213	0.69757	0.40926	0.02720
	0.975	0.99514	0.90739	0.72045	0.43977	0.06534
	0.990	0.99803	0.92512	0.74706	0.47525	0.10968
70	0.900	0.98162	0.86994	0.66429	0.36488	–0.02827
	0.950	0.98897	0.88622	0.68871	0.39745	0.01244
	0.975	0.99398	0.90035	0.70990	0.42570	0.04774
	0.990	0.99751	0.91677	0.73453	0.45854	0.08880
80	0.900	0.97991	0.86623	0.65872	0.35746	–0.03755
	0.950	0.98717	0.88146	0.68157	0.38793	0.00053
	0.975	0.99261	0.89467	0.70139	0.41435	0.03356
	0.990	0.99685	0.91004	0.72443	0.44507	0.07196
90	0.900	0.97842	0.86316	0.65411	0.35132	–0.04523
	0.950	0.98545	0.87752	0.67565	0.38004	–0.00933
	0.975	0.99109	0.88997	0.69434	0.40495	0.02181
	0.990	0.99600	0.90446	0.71606	0.43391	0.05802
100	0.900	0.97714	0.86056	0.65021	0.34612	–0.05173
	0.950	0.98390	0.87418	0.67065	0.37336	–0.01767
	0.975	0.98952	0.88600	0.68837	0.39700	0.01187
	0.990	0.99497	0.89974	0.70898	0.42448	0.04622

Table 4(b)
Recommended minimum values of $\hat{C}_a$ under accuracy requirement $C_0 = 0.50$ and various n, $1-\alpha$, $|\mu - m|/\sigma$ values for which the process is considered capable $(C_a \geqq C_0)$

n	$1-\alpha$	$\vert\mu - m\vert/\sigma$				
		0.25	0.50	0.75	1.00	1.25
10	0.900	0.99322	0.96683	0.87054	0.70263	0.47204
	0.950	0.99661	0.98291	0.91273	0.76007	0.54384
	0.975	0.99831	0.99138	0.94652	0.80988	0.60612
	0.990	0.99932	0.99654	0.97601	0.86764	0.67853
20	0.900	0.99347	0.94642	0.82621	0.64328	0.39785
	0.950	0.99673	0.96578	0.85668	0.68390	0.44863
	0.975	0.99836	0.98029	0.88310	0.71913	0.49266
	0.990	0.99935	0.99159	0.91380	0.76009	0.54387
30	0.900	0.99280	0.93349	0.80649	0.61699	0.36499
	0.950	0.99636	0.95005	0.83137	0.65015	0.40644
	0.975	0.99818	0.96429	0.85294	0.67892	0.44240
	0.990	0.99927	0.97995	0.87802	0.71237	0.48421
40	0.900	0.99171	0.92566	0.79474	0.60132	0.34539
	0.950	0.99573	0.94002	0.81628	0.63004	0.38130
	0.975	0.99784	0.95247	0.83496	0.65495	0.41244
	0.990	0.99914	0.96691	0.85669	0.68391	0.44864
50	0.900	0.99039	0.92031	0.78671	0.59062	0.33202
	0.950	0.99487	0.93315	0.80598	0.61631	0.36414
	0.975	0.99738	0.94430	0.82269	0.63859	0.39199
	0.990	0.99894	0.95725	0.84212	0.66450	0.42437
60	0.900	0.98903	0.91636	0.78079	0.58272	0.32215
	0.950	0.99382	0.92809	0.79838	0.60617	0.35147
	0.975	0.99676	0.93826	0.81364	0.62652	0.37689
	0.990	0.99869	0.95008	0.83137	0.65017	0.40646
70	0.900	0.98774	0.91329	0.77619	0.57659	0.31448
	0.950	0.99264	0.92415	0.79247	0.59830	0.34162
	0.975	0.99599	0.93357	0.80660	0.61713	0.36516
	0.990	0.99834	0.94451	0.82302	0.63903	0.39253
80	0.900	0.98660	0.91082	0.77248	0.57164	0.30830
	0.950	0.99144	0.92098	0.78771	0.59195	0.33369
	0.975	0.99507	0.92978	0.80092	0.60957	0.35571
	0.990	0.99790	0.94002	0.81629	0.63005	0.38131
90	0.900	0.98562	0.90877	0.76941	0.56754	0.30318
	0.950	0.99030	0.91835	0.78377	0.58669	0.32711
	0.975	0.99406	0.92665	0.79622	0.60330	0.34787
	0.990	0.99733	0.93630	0.81071	0.62261	0.37201
100	0.900	0.98476	0.90704	0.76681	0.56408	0.29885
	0.950	0.98926	0.91612	0.78043	0.58224	0.32155
	0.975	0.99302	0.92400	0.79225	0.59800	0.34125
	0.990	0.99665	0.93316	0.80599	0.61632	0.36415

Table 4(c)
Recommended minimum values of $\hat{C}_a$ under accuracy requirement $C_0 = 0.75$ and various n, $1-\alpha$, $|\mu - m|/\sigma$ values for which the process is considered capable $(C_a \geqq C_0)$

n	$1-\alpha$	$\lvert\mu - m\rvert/\sigma$				
		0.25	0.50	0.75	1.00	1.25
10	0.900	0.99661	0.98341	0.93527	0.85132	0.73602
	0.950	0.99831	0.99145	0.95636	0.88004	0.77192
	0.975	0.99915	0.99569	0.97326	0.90494	0.80306
	0.990	0.99966	0.99827	0.98800	0.93382	0.83927
20	0.900	0.99673	0.97321	0.91311	0.82164	0.69893
	0.950	0.99836	0.98289	0.92834	0.84195	0.72431
	0.975	0.99918	0.99014	0.94155	0.85957	0.74633
	0.990	0.99967	0.99580	0.95690	0.88005	0.77193
30	0.900	0.99640	0.96675	0.90325	0.80849	0.68249
	0.950	0.99818	0.97502	0.91568	0.82508	0.70322
	0.975	0.99909	0.98215	0.92647	0.83946	0.72120
	0.990	0.99963	0.98998	0.93901	0.85618	0.74210
40	0.900	0.99585	0.96283	0.89737	0.80066	0.67270
	0.950	0.99786	0.97001	0.90814	0.81502	0.69065
	0.975	0.99892	0.97624	0.91748	0.82747	0.70622
	0.990	0.99957	0.98345	0.92834	0.84196	0.72432
50	0.900	0.99520	0.96015	0.89336	0.79531	0.66601
	0.950	0.99743	0.96658	0.90299	0.80815	0.68207
	0.975	0.99869	0.97215	0.91135	0.81930	0.69599
	0.990	0.99947	0.97862	0.92106	0.83225	0.71219
60	0.900	0.99451	0.95818	0.89040	0.79136	0.66108
	0.950	0.99691	0.96404	0.89919	0.80309	0.67573
	0.975	0.99838	0.96913	0.90682	0.81326	0.68845
	0.990	0.99934	0.97504	0.91569	0.82508	0.70323
70	0.900	0.99387	0.95665	0.88810	0.78829	0.65724
	0.950	0.99632	0.96207	0.89624	0.79915	0.67081
	0.975	0.99799	0.96678	0.90330	0.80857	0.68258
	0.990	0.99917	0.97226	0.91151	0.81951	0.69627
80	0.900	0.99330	0.95541	0.88624	0.78582	0.65415
	0.950	0.99572	0.96049	0.89386	0.79598	0.66684
	0.975	0.99754	0.96489	0.90046	0.80478	0.67785
	0.990	0.99895	0.97001	0.90814	0.81502	0.69065
90	0.900	0.99281	0.95439	0.88470	0.78377	0.65159
	0.950	0.99515	0.95917	0.89188	0.79335	0.66356
	0.975	0.99703	0.96332	0.89811	0.80165	0.67394
	0.990	0.99867	0.96815	0.90535	0.81130	0.68601
100	0.900	0.99238	0.95352	0.88340	0.78204	0.64942
	0.950	0.99463	0.95806	0.89022	0.79112	0.66078
	0.975	0.99651	0.96200	0.89612	0.79900	0.67062
	0.990	0.99832	0.96658	0.90299	0.80816	0.68207

A useful decision-making procedure based on a recommended minimum value of $\hat{C}_a$ for judging whether the process meets the preset accuracy requirement is presented in the following.

Step 1. Decide the accuracy requirement C_0 $(0.25, 0.50, 0.75, \text{or } 1.00)$ and the confidence level $1-\alpha(0.900, 0.950, 0.975, \text{or } 0.990)$.

Step 2. Choose a random sample of size n from a stable process to calculate $\hat{\delta} = n|\bar{x}-m|^2/s^2$ and $\hat{C}_a = 1-(|\bar{x}-m|/d)$, where $s^2 = \sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)$.

Step 3. Running the SAS program displays in Appendix III (or referring to Tables 4(a)-4(c)) to obtain the recommended value $1-(1-C_0)$ $\times[\chi^2_\alpha(1,\hat{\delta})/\hat{\delta}]^{1/2}$.

Step 4. Conclude that the process is capable of the time with $100(1-\alpha)\%$ confidence if $\hat{C}_a > 1-(1-C_0)[\chi^2_\alpha(1,\hat{\delta})/\hat{\delta}]^{1/2}$. Otherwise, we do not have enough evidence to claim that the process is capable.

4. A demonstration example on process accuracy

To demonstrate how we may apply the proposed procedure to the actual data and judge whether the process is capable, we consider the data display in Table 5 (a total of 80 characteristic measurements are chosen from a stable process), which is a particular type of chip resistor used in the high/low frequency cross network design of the Home Theater Audio Video system. The upper specification limit (USL) and the lower specification limit (LSL) are set at 2.15Ω and 1.85Ω respectively with a Target value $T = m = 2.00\Omega$. Under the accuracy requirement $C_0 = 0.5$ with confidence level $1-\alpha = 0.95$, the sample data given that $\bar{x} = 1.9998$, $s = 0.0010$, $\hat{\delta} = 1.6215$, $\hat{C}_a = 0.99901$. Running the SAS program provided in Appendix III, we find that the recommended minimum value $1-(1-C_0)][\chi^2_\alpha(1,\hat{\delta})/\hat{\delta}]^{1/2} = 0.99888$. Since $\hat{C}_a > 1-(1-C_0)][\chi^2_\alpha(1,\hat{\delta})/\hat{\delta}]^{1/2}$, we claim that this process meets the accuracy requirement of the time with 95% confidence, and no further accuracy improvement action needs to be taken for the present.

5.. Conclusions

Process accuracy index C_a has been proposed in the manufacturing industry to provide numerical measures on assessing process performance with respect to the accuracy. Investigations on process centering

Table 5

Sample data of 80 characteristic measurements

2.0012	1.9994	2.000	1.9986	1.9989
2.0000	1.9974	1.9996	1.9990	2.0000
2.0015	1.9994	2.0002	2.0007	1.9995
2.0021	2.0002	1.9994	1.9996	2.0008
2.0003	2.0002	2.0008	1.9983	2.0011
1.9998	1.9984	1.9992	1.9995	1.9997
1.9985	2.0004	1.9995	1.9995	2.0000
2.0015	1.9982	1.9986	1.9982	1.9991
2.0000	2.0020	1.9987	2.0000	2.0011
2.0004	2.0000	2.0006	2.0017	1.9989
1.9995	1.9995	2.0004	1.9994	1.9991
2.0016	2.0004	1.9985	2.0003	2.0006
1.9997	1.9991	1.9997	1.9997	1.9999
1.9996	1.9999	2.0013	1.9999	2.0013
1.9987	2.0014	2.0006	2.0014	1.9991
1.9982	1.9994	1.9982	2.0017	1.9986

(the ability to cluster around the specification mid-point) in existing engineering statistics and quality assurance literature reveal conservatively rare. In this paper, a natural estimator of the accuracy index was considered under the normality assumption. The exact probability density function and the r-th moment raised from a non-central chi-squared distribution of the estimated accuracy index were derived. A decision-making procedure on process centering for in-plant applications was constructed. Values in lower bound of the estimated accuracy index required to ensure the process reaching a desirable accuracy level of the time were also tabulated. A practical example was presented to illustrate how the proposed reliable procedure may be applied.

Appendix I

Derivation of expression (8). Let the random variable $Y = \delta[(1-\hat{C}_a)/\times(1-C_a)]^2 = n[(\bar{X}-m)/\sigma]^2$ be distributed as $\chi^2(1,\delta)$, a non-central chi-squared distribution with one degree of freedom and non-centrality parameter $\delta = n[(\mu-m)/\sigma]^2 = 9n[C_p(1-C_a)]^2$. Then, the probability density function (pdf) of Y is given as

$$g(y) = \left\{\sum_{k=0}^{\infty} \frac{y^{k-(1/2)}\exp(-y/2)}{2^{k+(1/2)}\Gamma[k++(1/2)]} \times \frac{(\delta/2)^k \exp(-\delta/2)}{\Gamma(k+1)}\right\}, \quad 0 < y < \infty. \tag{A1}$$

Let the random variable $X = \hat{C}_a$, then $y = \delta[(1-x)/(1-C_a)]^2$ and $dy = -2\delta[(1-x)/(1-C_a)^2]dx$. The Jacobian of this transformation is $J = dy/dx = -2\delta[(1-x)/(1-C_a)^2$. Thus the pdf of $\hat{C}_a$. can be derived as $f(x) = g\{\delta[(1-x)/(1-C_a)]^2\}|J|$. The exact probability density function of $\hat{C}_a$ is presented in the following, where $-\infty < x \leqq 1$.

$$f(x) = \sum_{k=0}^{\infty}\left\{\left(\frac{\sqrt{2\delta}}{(1-C_a)\Gamma[k+(1/2)]}\right)\left[\frac{\delta}{2}\left(\frac{1-x}{1-C_a}\right)^2\right]^2 \times \exp\left[-\frac{\delta}{2}\left(\frac{1-x}{1-C_a}\right)^2\right]\frac{(\delta/2)^k\exp(-\delta/2)}{T(k+1)}\right\}. \quad \text{(A2)}$$

Appendix II

Derivation of expression (9). The r-th moment of $[(1-\hat{C}_a)/(1-C_a)]$ is

$$E\left(\frac{1-\hat{C}_a}{1-C_a}\right)^r = \int_{-\infty}^{1}\left(\frac{1-x}{1-C_a}\right)^r f(x)dx. \quad \text{(A3)}$$

Applying the technique on change of variables by setting $w = (\delta/2) \times[(1-x)/(1-C_a)]^2$, then $dx = -[(1-C_a)/\delta][(1-C_a)/(1-x)]dw = -[(1-C_a)/\sqrt{2\delta}](1/\sqrt{w})dw$. Therefore, the expression (A3) can be represented as

$$\begin{aligned} E\left(\frac{1-\hat{C}_a}{1-C_a}\right)^r &= \sum_{k=0}^{\infty}\left\{\left(\frac{(\sqrt{2/\delta})^r}{\Gamma[k+(1/2)]}\right)\left(\int_0^{\infty} w^{k+[(r-1)/2]}\exp(-w)dw\right) \times\left(\frac{(\delta/2)^k\exp(-\delta/2)}{T(k+1)}\right)\right\} \\ &= \sum_{k=0}^{\infty}\left\{\left(\frac{(\sqrt{2/\delta})^r}{\Gamma[k+(1/2)]}\right)\{\Gamma[k+(r+1)/2]\} \times\left(\frac{(\delta/2)^k\exp(-\delta/2)}{T(k+1)}\right)\right\}. \end{aligned} \quad \text{(A4)}$$

Therefore, the r-th moment of $1-\hat{C}_a$ is

$$E(1-\hat{C}_a)^r = \sum_{k=0}^{\infty}\left\{\left((1-C_a)\sqrt{\frac{2}{\delta}}\right)^r \times\left(\frac{\Gamma[k+(r+1)/2]}{\Gamma[k+(1/2)]}\right)\left(\frac{(\delta/2)^k\exp(-\delta/2)}{T(k+1)}\right)\right\}$$

$$= \sum_{k=0}^{\infty} \left\{ \left(\frac{1}{3C_p} \sqrt{\frac{2}{\delta}} \right)^r \left(\frac{\Gamma[k+(r+1)/2]}{\Gamma[k+(1/2)]} \right) \times \left(\frac{(\delta/2)^k \exp(-\delta/2)}{T(k+1)} \right) \right\}. \quad \text{(A5)}$$

Substituting $r = 1$ and $r = 2$ respectively into expression (A5), then we obtained

$$E(1-\hat{C}_a) = \sum_{k=0}^{\infty} \left\{ \left(\frac{1}{3C_p} \sqrt{\frac{2}{\delta}} \right) \left(\frac{\Gamma(k+1)}{\Gamma[k+(1/2)]} \right) \times \left(\frac{(\delta/2)^k \exp(-\delta/2)}{T(k+1)} \right) \right\}, \quad \text{(A6)}$$

and

$$\begin{aligned} E(1-\hat{C}_a)^2 &= \sum_{k=0}^{\infty} \left\{ \left(\frac{1}{3C_p} \sqrt{\frac{2}{\delta}} \right)^r \left(\frac{\Gamma[k+(3/2)]}{\Gamma[k+(1/2)]} \right) \times \left(\frac{(\delta/2)^k \exp(-\delta/2)}{T(k+1)} \right) \right\} \\ &= \sum_{k=0}^{\infty} \left\{ \left(\frac{2k+1}{9nC_p^2} \right) \left(\frac{(\delta/2)^k \exp(-\delta/2)}{T(k+1)} \right) \right\}. \quad \text{(A7)} \end{aligned}$$

Applying the following facts that $E(\hat{C}_a) = 1 - E(1-\hat{C}_a)$ and $E(\hat{C}_a)^2 = E(1-C_a)^2 - 2E(1-\hat{C}_a) + 1$, then the first two moments of $\hat{C}_a$ can be derived as

$$E(\hat{C}_a) = 1 - \sum_{k=0}^{\infty} \left\{ \frac{1}{3C_p} \sqrt{\frac{2}{n}} \frac{\Gamma(k+1)}{\Gamma[k+(1/2)]} \frac{(\delta/2)^k \exp(-\delta/2)}{\Gamma(k+1)} \right\}, \quad \text{(A8)}$$

$$E(\hat{C}_a^2) = 1 + \sum_{k=0}^{\infty} \left\{ \left[\frac{2k+1}{9nC_p^2} - \frac{1}{2C_p} \sqrt{\frac{2}{n}} \frac{\Gamma(k+1)}{\Gamma[k+(1/2)]} \right] \times \frac{(\delta/2)^k \exp(-\delta/2)}{\Gamma(k+1)} \right\}. \quad \text{(A9)}$$

Appendix III

```
/* Recommended minimum values for accuracy index */;
OPTIONS REPLACE PAGESIZE=58 LINESIZE=78 NODATE; DATA LOW;
  ALPHA=0.050; C0=0.50;
  D1=0.25; D2=0.50; D3=0.75; D4=1.00; D5=1.25;
```

```
DO N=10 TO 100 BY 10;
  LOW1=1-(1-C0)*(CINV(ALPHA,1,N*D1**2)/(N*D1**2))**0.5;
  LOW2=1-(1-C0)*(CINV(ALPHA,1,N*D2**2)/(N*D2**2))**0.5;
  LOW3=1-(1-C0)*(CINV(ALPHA,1,N*D3**2)/(N*D3**2))**0.5;
  LOW4=1-(1-C0)*(CINV(ALPHA,1,N*D4**2)/(N*D4**2))**0.5;
  LOW5=1-(1-C0)*(CINV(ALPHA,1,N*D5**2)/(N*D5**2))**0.5;
  OUTPUT;
END;
FORMAT LOW1 LOW2 LOW3 LOW4 LOW5 6.4;
PROC PRINT DATA = LOW;
VAR N LOW1 LOW2 LOW3 LOW4 LOW5;
RUN;
```

References

[1] V. E. Kane (1986), Process capability indices, *Journal of Quality Technology*, Vol. 18 (1), pp. 41–52.

[2] W. L. Pearn, G. H. Lin and K. S. Chen (1998), Distributional and inferential properties of the process precision and process accuracy indices, *Communications in Statistics: Theory and Methods*, Vol. 27 (4), pp. 985–1000.

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