Optimal production and inspection strategy while considering preventive maintenance errors and minimal repair

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Abstract

This paper extends an integrated model of *Economic Production Quantity* (EPQ) and *Preventive Maintenance* (PM) to incorporate possibilities of PM errors and minimal repair. Our model determines simultaneously the optimal number of inspection, the duration of the first inspection interval, the EPQ and the PM level. Numerical examples of Weibull shock models are given to show that incorporating PM errors will raise the expected total cost and lower the EPQ while allowing minimal repair will lower the expected total cost. Our analyses demonstrate that both PM errors and minimal repair significantly influence the optimal policy and the expected cost.

Keywords and phrases : *Production economics, preventive maintenance, inspection planning, operations strategy.*

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1. Introduction

The classical *Economic Production Quantity* (EPQ) model assumes that the production system is free of failures and that all items produced are perfect [13]. Many research efforts have been made to extend the classical EPQ model by relaxing various assumptions. In real world situation, for example, although the production process begins by producing lots in an 'in-control' state, it may become 'out-of-control' producing defective items. Rosenblatt and Lee [11] considered the effects of process deterioration on the optimal EPQ. They assumed that the time at which the process shifts from the in-control state to the out-of-control state follows an exponential distribution and found that the resulting EPQ is smaller than that of the classical model because the use of smaller lots produces fewer defective items. Porteus [9] obtained similar results. Hariga and Ben-Daya [5] extended the model to consider the case where the deterioration of the process follows a general distribution.

The modeling of production systems must also address the maintenance aspect. Proper *Preventive Maintenance* (PM) is essential for improving production processes and thus preventing system failure. Most PM models assume that the system is 'as good as new' after each PM action. However, a more realistic approach may model the failure rate of the system after PM as being somewhere between 'as good as new' and 'as bad as old'. Nakagawa [7], and Pham and Wang [8] reviewed the various optimal strategies under imperfect maintenance. Basically there are two approaches for modeling imperfect PM: (1) PM reduces the hazard rate while the number of performing PM increases, and (2) PM reduces aging of the production system. The change in the aging of the system will have an effect on the distribution of the elapsed time of the system shifting to the out-of-control state.

Tseng *et al* [14] considered a model of imperfect maintenance using a PM that either restores the system to the 'as good as new' state or the system will fail immediately thereafter because of faulty maintenance. In general, a system after maintenance is not as good as new, but becomes as if it were younger than its actual age. Rahim [10] explored the optimal control chart design parameter and the EPQ for an imperfect production process. Ben-Daya and Makhdoum [2] discussed the impact of various PM policies when the EPQ model is combined with control chart design.

In a ground-breaking effort, Ben-Daya [3, 4] presented an integrated model for the joint determination of EPQ and PM level thereby capturing the underlying relationship among production maintenance and quality. His model considered the optimal inspection interval, inspection frequency and production quantity for the general deterioration distribution with rapidly increasing failure rate. Lin [6] extended Ben-Daya's model to allow for errors in PM due to human or technological limitation and derived optimal solution for production quantity and inspection points in relation to PM errors. Sheu and Chen [12], on the other hand, extended Ben-Daya's model to include the possibility of 'minimal repair'. It should be noted that the production cycle is rarely interrupted in practice even when the system is in an out-of-control state. In Sheu and Chen's model, the out-of-control state consists of two types: a minimal repair can remove type I out-of-control state and the production system will not be interrupted, whereas type II out-of-control state makes the production system stop and restoration work is carried out. In this paper, we combine Lin's and Sheu and Chen's extensions together to incorporate both PM errors and minimal repair. Our integrated model determines simultaneously the optimal number of inspections, the duration of the first inspection interval, the EPQ and the PM level while allowing possibilities of PM errors and minimal repair. Numerical examples are presented to illustrate important aspects of the proposed model. In particular, it is shown that incorporating PM errors raises the expected total cost and lowers the EPQ while allowing minimal repair leads to lower expected total cost due to reductions in quality-related costs. Our analysis thus demonstrate that both PM errors and minimal repair significantly influence the optimal policy and the expected cost.

The rest of the paper is organized as follows. Section 2 presents the model and its mathematical formulation. Section 3 derives the optimal solution. Illustrative examples are given in Section 4 with numerical solutions. Section 5 contains some concluding remarks. Section 6 gives the proofs of the properties.

2. Model development

2.1 *Assumptions*

Consider a production process producing a single product. The process is in either the *in-control* state or the *out-of-control* state.

At the beginning of a production cycle, the system is assumed to be in an in-control state producing items of good quality. However, the process may shift to an out-of-control state and be monitored periodically by inspections. If the system is inspected and judged to be in control, PM is implemented. The system failure rate will decrease given correctly implemented PM; but an incorrectly implemented PM will cause the system to enter an out-of-control state. The reduction in the effective age of the system depends on the level of PM performed. The PM level, therefore, is a decision variable. On the other hand, if the system is inspected and judged to be out of control, our model allows two types of out-of-control states: (1) For type I out-of-control state, a minimal repair is performed to restore the system to the in-control state, and the system failure rate remains unchanged. (2) For type II out-of-control state, a minimal repair is not sufficient — the production has to cease and the system has to be restored to the in-control sate and to the 'as good as new' condition by a complete repair or replacement if necessary.

The elapsed time for the process to be in the in-control state is a random variable assumed to follow a general distribution with increasing hazard rate. The process is inspected at times $t_1, t_2, \ldots, t_k$ to assess its state. For simplicity, we assume inspection and PM times are negligible. The productions cycle ends either when the system shifts to type II out-of-control state or after the k th inspection intervals whichever occurs first. Each inspection interval has the same cumulative hazard rate.

2.2 *Notation*

To facilitate comparison, we shall use the same notations as in Ben-Daya [4], Lin [6], and Sheu and Chen [12]:

D	demand rate in units per unit of time
P	production rate in units per unit of time, where $(P > D)$
T	actual production time for each production cycle
S	setup cost for each production cycle
C_h	holding cost per unit per unit of time
C_I	inspection cost per unit
θ	probability of type II out-of-control state when the system is out-of-control
η	imperfectness factor showing the effect of PM on the age of the system

γ_k	imperfectness coefficient at the kth PM
d_I	percentage of defective units produced when the process is in type I out-of-control state
d_{II}	percentage of defective units produced when the process is in type II out-of-control state
C_d	cost incurred by producing a defective item
ϕ_j	restoration delay during interval j
$R(\phi)$	restoration cost
k	number of inspections conducted during each production cycle
h_j	duration of the jth inspection interval
t_j	time of the jth PM, $t_j = \sum\limits_{i=1}^{j} h_i$
N_j^I	number of defective items produced in (t_{j-1}, t_j) under type I out-of-control state, $j = 1, 2, \ldots, k$
N_j^{II}	number of defective items produced in (t_{j-1}, t_j) under type II out-of-control state, $j = 1, 2, \ldots, k$
b_i	actual age of the system immediately before the ith PM
a_i	actual age of the system immediately after the ith PM
$f(t)$	probability density function of the time to shift distribution
$F(t)$	cumulative distribution
$r(t)$	hazard function
C_{apm}	cost of the actual PM activities
C_{mpm}	cost of the maximum PM level
C_{mr}	cost of performing minimal repair
δ	probability of committing PM errors
P_j	conditional probability that the process shifts into the out-of-control state during the time interval (t_{j-1}, t_j) given that it was in the in-control state at time t_{j-1}

2.3 *The production-inventory cycle*

The methodology adopted in this paper involves a number of steps. First, the equations for the production cycle and the inventory cycle are specified. Next, these equations are solved to formulate the cost model. Finally, a heuristic numerical algorithm is proposed to search for the minimum-cost solution. The details of this methodology are discussed below.

Property 1. The expected production cycle is given by

$$E(T) = \sum_{j=1}^{k} h_j (1-\delta)^{j-1} \prod_{i=1}^{j-1} (1-\theta p_i)\,, \tag{1}$$

and the expected inventory cycle length is give by (Figure 1)

$$E(CT) = \frac{P}{D} \times E(T)\,. \tag{2}$$

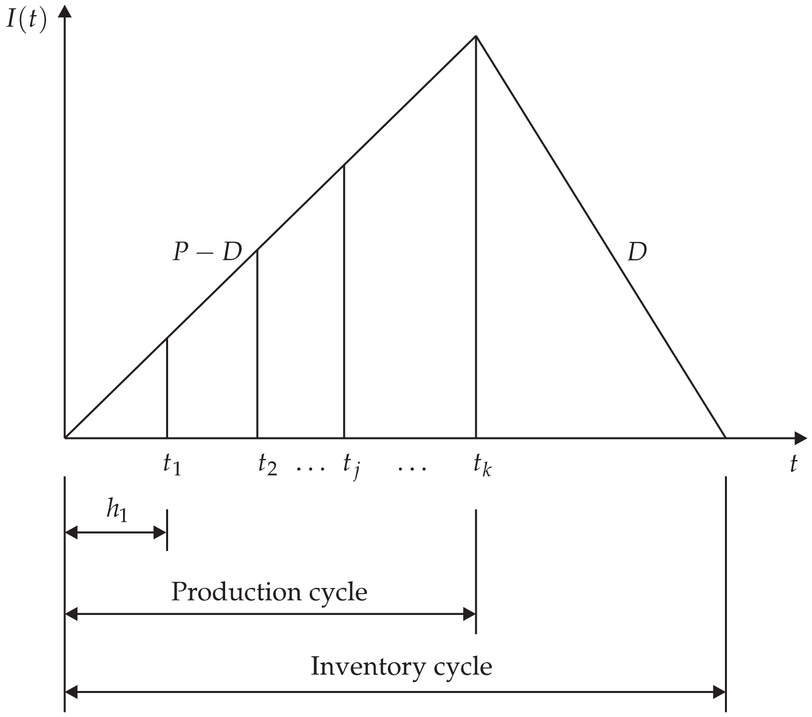

Figure 1
Inventory cycle

2.4 *The cost components*

The expected total cost per cycle is given by

$$ETC = \frac{S + E(HC) + E(PM) + E(IC) + E(DC) + E(RC)}{E(CT)}\,, \tag{3}$$

where S, $E(HC)$, $E(PM)$, $E(IC)$, $E(DC)$ and $E(RC)$ are the setup cost, inventory holding cost, PM and minimal repair cost, inspection cost, cost of producing defective items and restoration cost, respectively.

The production cost for each product is fixed and does not need to be considered. The inventory holding cost is

$$E(HC) = \left(\frac{C_h}{2}\right) E(T)^2 (P - D)\frac{P}{D}\,. \tag{4}$$

After each PM, the system will not be as good as new, but will be 'younger' with the age depending on the level of PM activities. The reduction in the age of the equipment is a function of the cost of PM. Let

$$\gamma_k = \eta^{k-1}\frac{C_{apm}}{C_{mpm}}\,, \tag{5}$$

where $0 \le \eta \le 1$. The parameter η is a degradation factor which shows the effect of PM on the age of the system. Ben-Daya [3] considered linear and nonlinear relationship between age reduction and PM cost. Here, we assume relationship is linear and given by

$$a_k = (1 - \gamma_k) b_k. \tag{6}$$

Note that the effective age of the equipment at time t_j is given by

$$\begin{aligned} b_1 &= h_1\,, \\ b_j &= a_{j-1} + h_j\,, \qquad j = 2, 3, \ldots, k\,. \end{aligned} \tag{7}$$

This change in the age of the equipment due to PM will affect the number of defective items, restoration cost and the length of the production run and hence allows the joint optimization of production quantity, quality control costs and the cost of PM. Thus production, quality and maintenance aspects are integrated in one model.

Since the inspection is error free and PM activities are carried out after each inspection except that the production system has to stop, the expected PM and minimal repair cost per production cycle is give by

$$\begin{aligned} E(PM) = C_{apm}\Bigg\{ &\sum_{j=1}^{k-1}\prod_{i=1}^{j}[(1-\theta p_i)(1-\delta)] \\ &+ \sum_{j=1}^{k-1}(1-\theta p_j)\delta\prod_{i=1}^{j-1}[(1-\theta p_i)(1-\delta)]\Bigg\} \\ &+ C_{mr}(1-\theta)\sum_{j=1}^{k-1}(1-\delta)^{j-1} p_j \prod_{i=1}^{j-1}(1-\theta p_i)\,. \end{aligned} \tag{8}$$

The proof of (8) is similar to that of Property 1. A PM is performed after each inspection and no PM is being performed at the end of the production cycle. Hence the inspection cost is

$$\begin{aligned} E(IC) \;=\; & C_I\Big\{1+\sum_{j=1}^{k-1}\prod_{i=1}^{j}[(1-\theta p_i)(1-\delta)] \\ & +\sum_{j=1}^{k-1}(1-\theta p_j)\delta\prod_{i=1}^{j-1}[(1-\theta p_i)(1-\delta)]\Big\}. \end{aligned} \tag{9}$$

Let the $E(N_j^I)$ be the expected number of defective item s due to type I out-of-control state during the j th inspection interval, then $E(N_j^I)$ can be given by the following formula:

Property 2.

$$E(N_j^I)=\int_{a_{j-1}}^{b_j} d_I P(b_j-t)\frac{(1-\theta)f(t)[\bar{F}(t)]^{-\theta}}{[\bar{F}(a_{j-1})]^{1-\theta}}dt\,. \tag{10}$$

Similarly, the expected number of defective items due to type II out-of-control state during the j th inspection interval can be given by:

$$E(N_j^{II})=\int_{a_{j-1}}^{b_j} d_{II} P(b_j-t)\frac{\theta f(t)[\bar{F}(t)]^{\theta-1}}{\bar{F}(a_{j-1})]^{\theta}}dt\,. \tag{11}$$

The total expected number of defective items per production cycle is

$$\begin{aligned} E(N) \;=\; & \sum_{j=1}^{k}(1-\delta)^{j-1}p_j[(1-\theta)E(N_j^I)+\theta E(N_j^{II})] \\ & \times\prod_{i=1}^{j-1}(1-\theta p_i)\,. \end{aligned} \tag{12}$$

The total cost of producing defective items per cycle is

$$\begin{aligned} E(DC) \;=\; & C_d\sum_{j=1}^{k}(1-\delta)^{j-1}p_j[(1-\theta)E(N_j^I)+\theta E(N_j^{II})] \\ & \times\prod_{i=1}^{j-1}(1-\theta p_i)\,. \end{aligned} \tag{13}$$

Assume that the restoration cost change linearly with the detection delay

$$R(b_j-t)=r_0+r_1(b_j-t)\,, \tag{14}$$

where r_0 and r_1 are some constants. The restoration cost during the j th interval is given by

$$\begin{aligned} E(RC_j) &= \int_{a_{j-1}}^{b_j} R(b_j-t)\frac{\theta f(t)[\bar{F}(t)]^{\theta-1}}{\bar{F}(a_{j-1})]^{\theta}}dt \\ &= (r_0+r_1b_j)\left[1-(\frac{\bar{F}(b_j)}{\bar{F}(a_{j-1})})^{\theta}\right]-r_1\int_{a_{j-1}}^{b_j} t\frac{\theta f(t)[\bar{F}(t)]^{\theta-1}}{[\bar{F}(a_{j-1})]^{\theta}}dt. \end{aligned} \tag{15}$$

Therefore, the expected restoration cost per cycle is

$$E(RC) = \theta \sum_{j=1}^{k} (1-\delta)^{j-1} p_j \prod_{i=1}^{j-1} (1-\theta p_i) \Bigg\{ (r_0 + r_1 b_j) \left[1 - \left(\frac{\bar{F}(b_j)}{\bar{F}(a_{j-1})} \right)^{\theta} \right] \\ - r_1 \int_{a_{j-1}}^{b_j} t \frac{\theta f(t)[\bar{F}(t)]^{\theta-1}}{[\bar{F}(a_{j-1})]^{\theta}} dt \Bigg\}. \qquad (16)$$

3. Solution algorithm

3.1 *Optimal solution*

The optimal solution is the strategy with the minimum expected total cost. Following Banerjee and Rahim [1], we let each inspection interval have the same cumulative hazard rate

$$\int_{t_j}^{t_{j+1}} r(t)dt = \int_0^{t_1} r(t)dt. \qquad (17)$$

Since the hazard rate is reduced at the end of each interval because of PM, condition (17) becomes

$$\int_{a_{j-1}}^{b_j} r(t)dt = \int_0^{h_1} r(t)dt, \qquad j = 2, 3, \ldots, k. \qquad (18)$$

If the time during which the process remains in the in-control state follows a Weibull distribution, i.e. its probability density function is given by

$$f(t) = \lambda \nu t^{\nu-1} e^{-\lambda t^{\nu}}, \qquad t > 0,\ \nu \geq 1,\ \lambda > 0.$$

then using (18), the duration of the inspection intervals $h_j, j = 2, 3, \ldots, k$, can be determined recursively as follows:

$$h_j = [(a_{j-1})^{\nu} + h_1^{\nu}]^{1/\nu} - a_{j-1}, \qquad j = 2, 3, \ldots, k. \qquad (19)$$

Note that if we make the assumptions $\delta = 0$, $\theta = 1$ and $k = 1$, then our model becomes essentially the classical EPQ model. Ben-Daya [4] is a special case of our model with $\delta = 0$ and $\theta = 1$, while Sheu and Chen [12] is the case with $\delta = 0$, and Lin [6] with $\theta = 1$.

3.2 *Algorithm description*

The solution procedure is reduced to determining the values of three decision variables k, h_1 and C_{apm}. Since no general explicit solution can be derived for those decision variables, we have to employ a stepwise partial enumeration procedure to minimize the cost function. However, due to the characteristics of the cost function, some modifications on the standard method have to be made to account for the inherent integrality constraint on the number of inspections.

The optimal value of $k \geq 2$ could be determined by the following two constraints:

$$ETC(k-1) \geq ETC(k) \quad \text{and} \quad ETC(k+1) \geq ETC(k).$$

Therefore, the optimal value k^* and h_1^* can be obtained by the following procedure if we have determined the level of PM:

Step 1. Estimate k_0, the maximum number of inspections undertaken during each production cycle, either from historical records or from the condition of production.

Step 2. We begin by setting $k = 1$ and search for the optimal value h_1 and the expected total cost ETC_1.

Step 3. By repeating *Step* 2 for $k = 2, 3, \ldots, k_0$, we can calculate optimal h_1 for each different k and the corresponding ETC_2, $ETC_3, \ldots, ETC_{k_0}$.

Step 4. The optimal values h_1^* and k^* satisfy the following condition:

$$ETC(h_1^*, k^*; C_{apm}) = \text{Min}\{ETC_j, j = 1, 2, \ldots, k_0\}.$$

4. Numerical analyses

We present a numerical example to illustrate important aspects of the developed model. The process shift mechanism is assumed to follow a Weibull distribution. Weibull scale and shape parameters are $\lambda = 5$, $\nu = 2.5$, respectively. We consider four different probabilities of committing PM errors $(\delta = 0, \delta = 0.01, \delta = 0.05, \delta = 0.1)$ in our computations. The following data are used for other parameters: $D = 500$, $P = 1000$, $C_h = \$0.5$, $S = \$150$, $C_d = \$20$, $C_{mpm} = \$20$, $C_I = \$10$, $r_0 = \$10$, $r_1 = \$0.15$, $C_{mr} = \$10$, $\eta = 0.99$, $d_I = 0.2$ and $d_{II} = 0.4$.

The results for different PM levels and different probabilities of committing PM errors were simulated and summarized in Table 1. These results indicate clearly that as the PM level increases, the expected total cost decreases. For example, in the case of zero probability of PM error $(\delta = 0)$, the expected total cost is \$290.32 if no PM is performed, but only \$249.70 if maximum PM is performed. The reduction in the expected total cost ($40.62) greatly exceeds the cost of performing PM ($20). Note also that the ETC increases as the probability of PM errors increases. But when C_{apm} equals C_{mpm}, the expected cost is uniformly at its minimum. That means that if we want to perform PM, we should perform at the maximum level.

Table 1

Effect of the PM level on total cost [a]

	$\frac{C_{apm}}{C_{mpm}}$				
ETC	$= 0.0$	$= 0.25$	$= 0.05$	$= 0.75$	$= 1.0$
$\delta = 0.00$	290.32	279.84	267.49	256.78	249.70
$\delta = 0.01$	291.00	280.58	268.20	257.36	250.06
$\delta = 0.05$	293.77	283.61	271.15	259.82	251.70
$\delta = 0.1$	297.32	287.56	275.11	263.26	254.21

[a] $k = 3$, $\theta = 0.5$, $h_1 = 0.2635$

Assuming then that maximum PM is performed, we calculate and summarize in Table 2 the optimal number of inspections, the optimal duration of the first inspection interval, the EPQ and the expected total cost under various conditions. We have the following observations: (1) Incorporating PM errors would raise the duration of the first inspection interval and the expected total cost, lower the EPQ, but have no effect on the number of inspections. (2) The possibility of minimal repair would indeed lower the expected total cost. For instance, in the case of no PM errors $(\delta = 1)$, the expected total cost would be $262.81 when there is no possibility of minimal repair $(\theta = 1)$, but would be reduced to $249.70 with the possibility of minimal repair $(\theta = 0.5)$. Our numerical simulations thus demonstrate that the possibilities of both PM errors and minimal repair would have significant effect on the expected total cost.

Figure 2 shows the effects of minimal repair on the expected cost: the closer θ is to 0.5, the lower the expected cost. Figure 3 shows the adverse effects of PM errors on the expected cost. Figure 4 illustrates the combined influences of minimal repair and PM errors: the expected cost is lowest when $\theta = 0.5$ and $\delta = 0$.

Table 2

Optimal solutions under various conditions

	$\theta = 1.0$				$\theta = 0.5$				$\theta = 0.0$			
	k^*	h_1^*	Q^*	ETC	k^*	h_1^*	Q^*	ETC	k^*	h_1^*	Q^*	ETC
$\delta = 0.000$	4	0.2198	743	262.81	3	0.2625	723	249.70	3	0.2478	741	252.96
$\delta = 0.005$	4	0.2203	739	263.02	3	0.2630	721	249.88	3	0.2483	739	253.14
$\delta = 0.01$	4	0.2209	735	263.23	3	0.2635	719	250.06	3	0.2489	737	253.32
$\delta = 0.05$	4	0.2249	704	265.18	3	0.2678	701	251.64	3	0.2532	720	254.89
$\delta = 0.1$	4	0.2297	667	268.23	3	0.2730	679	253.91	3	0.2587	699	257.14

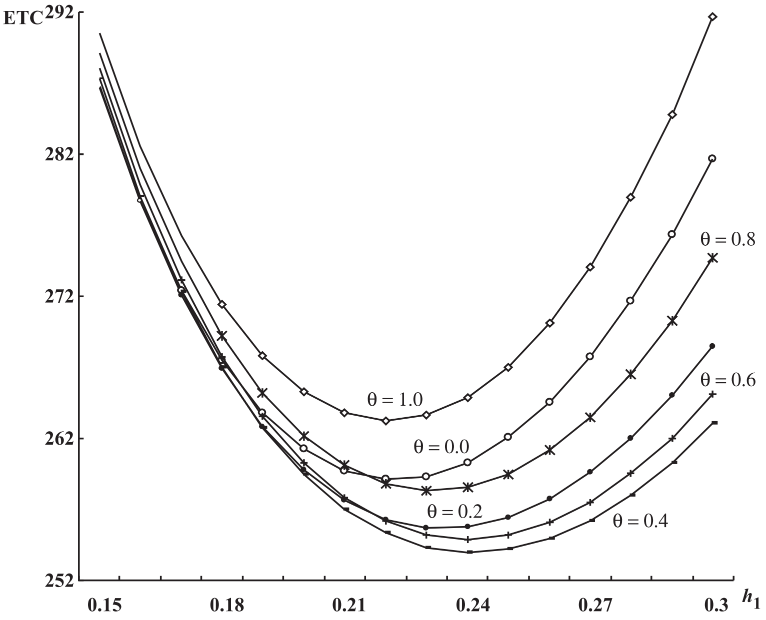

Figure 2

Total cost curves for different probabilities of type I out-of-control state with minimal repair $(k = 4, \delta = 0.01)$

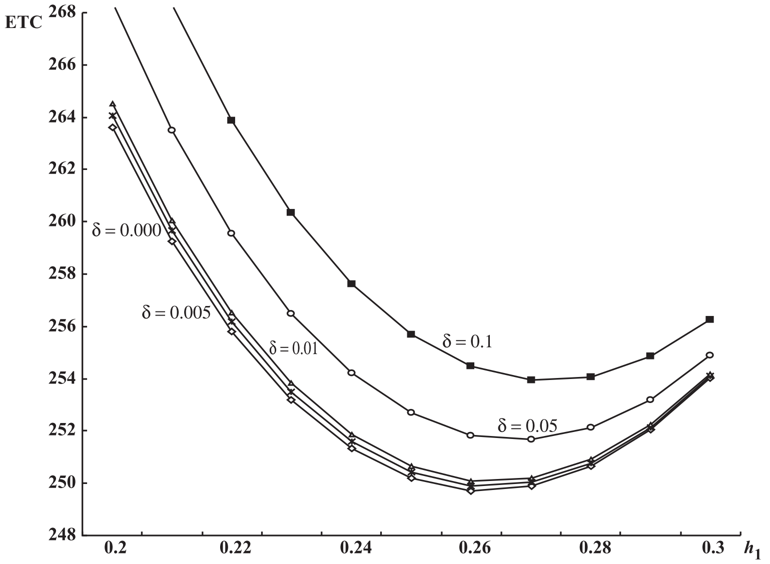

Figure 3

Total cost curves for different probabilities of PM errors

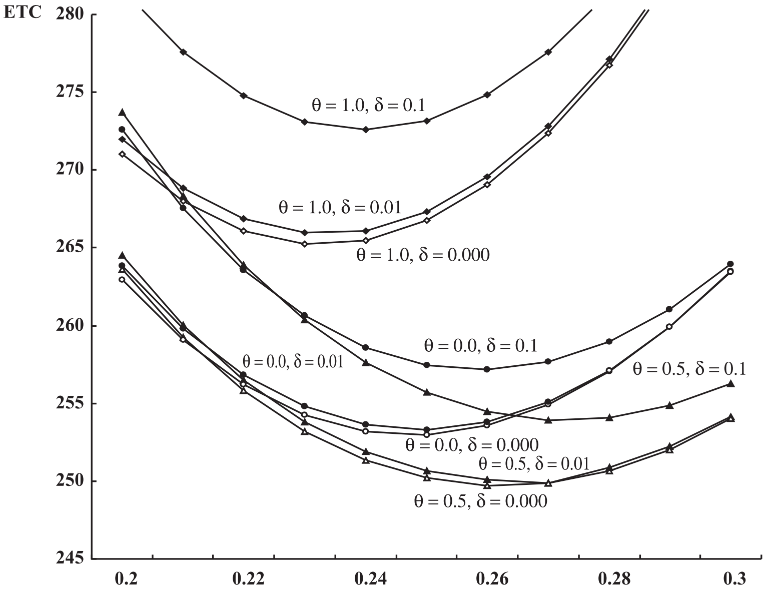

Figure 4

Total cost curves for different combined probabilities of PM errors and minimal repair $(k = 3)$

5. Conclusions

The main contribution of this paper is to extend Ben-Daya's [4] integrated model of EPQ and PM to incorporate the possibilities of PM errors and minimal repair. These generalizations are important because in many real world situations the out-of-control state can be removed by a minimal repair and there may be maintenance errors due to human or technological limitations. Our model determines simultaneously the optimal number of inspections, the duration of the first inspection interval, the EPQ and the PM level.

Numerical solutions can be easily generated using a simple electronic spreadsheet and requires a short run time on a personal computer. The results show that incorporating PM errors will raise the expected total cost and lower the EPQ while allowing minimal repair will lower the expected total cost. Our analyses thus demonstrate that the possibilities of both PM errors and minimal repair significantly influence the optimal policy and the expected total cost. The results obtained for different PM levels indicate that as the PM level increases, the expected cost decreases. If we want to perform PM, we should perform at the maximum level.

6. Proof of properties

***Proof of Property* 1.** Let $E(T_j)$ be the expected residual time in the production cycle beyond time t_j given that the system is in an in-control or a type I out-of-control state and that PM has been performed correctly at time t_j, $E(T_0) = E(T)$. Let p_j be the conditional probability that the process shifts to the out-of-control state during the time interval (t_{j-1}, t_j) given that the process was in control at time t_{j-1}, then $p_j = (F(b_j) - F(a_{j-1}))/\bar{F}(a_{j-1})$.

In order to find the expression for $E(T)$, let us consider the possible states of process at the end of the first interval (i.e., at $t_1 = h_1$). For each possible state, the expected residual time in the production cycle and the associated probabilities are as in the following table.

Therefore,

$$E(T) = h_1 + [(1 - \theta p_1)(1 - \delta)]E(T_1).$$

Similarly, for $j = 1, 2, \ldots, k - 1$, we have

$$E(T_j) = h_{j+1} + [(1 - \theta p_{j+1})(1 - \delta)]E(T_{j+1}).$$

State	Probability	Expected residual time
In-control state with correct PM	$(1-p_1)(1-\delta)$	$E(T_1)$
In-control state with incorrect PM	$(1-p_1)\delta$	0
Type I out-of-control state with correct PM	$p_1(1-\theta)(1-\delta)$	$E(T_1)$
Type I out-of-control state with incorrect PM	$p_1(1-\theta)\delta$	0
Type II out-of-control state	$p_1\theta$	0

Note that

$$E(T_k) = 0\,,$$

therefore,

$$E(T) = \sum_{j=1}^{k} h_j(1-\delta)^{j-1}\prod_{i=1}^{j-1}(1-\theta p_i)\,.$$

***Proof of Property* 2.** Let Y_A be the time to type I out-of-control state of a process with age A. Let g_A, G_A and $\bar{G}_A$ be the probability density function, the cumulative density function and the survival function, respectively, of Y_A . Then

$$\begin{aligned}
\bar{G}_A(t) &= \Pr(Y_A > t) = e^{-\int_0^t (1-\theta)r(A+u)du} = \left[\frac{\bar{F}(A+t)}{\bar{F}(A)}\right]^{1-\theta}\,,\\
G_A(t) &= 1-\bar{G}_A(t) = 1-\left[\frac{\bar{F}(A+t)}{\bar{F}(A)}\right]^{1-\theta}\,,\\
g_A(t) &= \frac{(1-\theta)f(A+t)[\bar{F}(A+t)]^{-\theta}}{[\bar{F}(A)]^{1-\theta}}\,.
\end{aligned}$$

The expected number of defective items produced under type I out-of-control state during the j th interval can be derived as in the following:

$$\begin{aligned}
E(N_j^I) &= \int_0^{h_j} d_I P(h_j-t) g_{a_{j-1}} dt\\
&= \int_0^{h_j} d_I P(h_j-t)\frac{(1-\theta)f(a_{j-1}+t)[\bar{F}(a_{j-1}+t)]^{-\theta}}{[\bar{F}(a_{j-1})]^{1-\theta}}dt\\
&= \int_{a_{j-1}}^{b_j} d_I P(h_j-t)\frac{(1-\theta)f(a_{j-1}+t)[\bar{F}(a_{j-1}+t)]^{-\theta}}{[\bar{F}(a_{j-1})]^{1-\theta}}dt\,.
\end{aligned}$$

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