A two phase local global search algorithm using new global search strategy

Djalil Boudjehem [1,2,*]

Nora Mansouri [2,†]

[1] *LAIG Laboratoire d'Automatique et Informatique de Guelma*
BP401, Guelma 24000
Algérie

[2] *Département d'électroniquer*
Faculté des sciences de l'ingénieur
Université Mentouri Constantine
Route de Ain-El-Bey
25000 Constantin
Algérie

Abstract

In this paper, we present a two phase local global search algorithm that is used to remedy the problems associated to the presence of sensitive local optima. However, The presence of such optima in most optimization problems make the global optimization very difficult in the sense that, as soon as the design space exhibits such local optima, the optimization method falls inside and are unable to leave it to a potentially better region. To accurate this problem we propose a new global search technique, which is called *Circular Design*. We propose also a new point scattering design and a new population evolution scheme the new algorithm works on the principal of evaluating a set of super individuals only. The local search is invoked at each time where a reallocation of the center of the Circular Design is needed, and it has the ability of significantly enlarge the attraction basin of the global optimum in order to reduce the probability of a possible convergence to an interesting local optimum. To illustrate the effectiveness of the proposed algorithm, numerical applications are performed with different benchmark problems; and the obtained results are satisfactory in terms of the solution quality and the time need to reach the global optimum.

Keywords : *Space partition, basins of attraction, circular design, local search, region potential.*

E-mail: dj_boudjehem@yahoo.fr

†*E-mail*: nor_mansouri@yahoo.fr

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1. Introduction

Finding the global minimum of a non convex function is the most important objective in global optimization, especially for problems with multiple optima. The difficulty of this operation has for a long time, leads to develop many algorithms and techniques to deal with most difficult global optimization problems [1], [2], [3].

The efficiency of all these algorithms is heavily dependent on the choice of initial parameters or population and there is no technique thoroughly reliable in terms of solution quality.

The classical techniques also called gradient methods base their search on the calculation of the first or second derivative of the cost function. Their convergence, to the global optimum is guaranteed only in the case where the initial algorithm values are chosen in the neighborhood of the solution.

Techniques such as evolutionary algorithms or simulated annealing are developed under different principles and are inspired from different sources. Each technique has its own properties, advantages and disadvantages. For example, evolutionary algorithms draw their inspiration from the hypothesis that the species evolve through a process of survival of the fittest individuals, whereas the simulated annealing method is slow and must be adapted with the problem being treated. The use of evolutionary algorithm requires very intensive computation and the performances depend on the choice of genetic initial population and the performances of simulated annealing depend on the problem.

To remedy to all these inconvenient, adaptive partitioning algorithms (APA) have represented an issue to find the best initial values for iterative optimization algorithms. These algorithms operate on the principle of consecutive search space reducing and resampling. After several reductions, the subspace most susceptible to contain the global optimum is selected, many works have been published on the subject [4], [5], [6], [7] and [8].

In this paper we propose a new approach that tries to find the global optimum with a minimum number of calculations and with high solution quality taking into account time consumption and precision. The main idea of the proposed technique is a global probabilistic search around the super individuals of a population candidate solution associate to a local search. The combination of the local search with the global search increases may ensure very interesting convergence time. In this approach we base our search only on the information given by the set of the Super Individuals. We use the Circular Design technique in the global search phase and reallocate the center of each sub-region by a local search. To illustrate the effectiveness of the proposed algorithm, numerical applications are performed with different test functions.

2. Local search

Definition (Attraction Basins). The attraction basins of a local optimum X_i^* is the set of the points $X_1, \ldots, X_k$ of the search space E such that for every point X_j, $l < j < k'$, that is the start point of any local search algorithm will converge to the local optimum X_i^*, and the normalized size of the attraction basin is $k'/|E|$.

A search is said to be a local search if it can only finds the nearest local optimum within the boundaries of its starting point. Today, deal with hard optimization problems, shows that a two-phase global/local search can be used to bring out with better results.

The use of the local search with the global search strategy to change the landscape is very useful to localize the global optimum. Figure 1 shows the effect of the local search on the landscape of the objective function, which results in making the global optimization easier, by enlarging the attraction basin of the global optimum and minimizing the effect of all sensitive local optima. At this level we can detect easily the attraction basin of the global optimum.

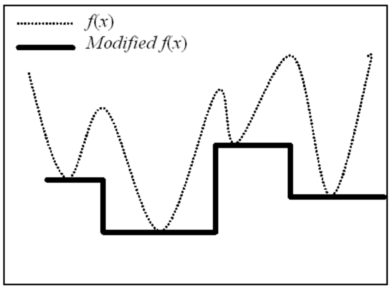

Figure 1
The effect of a local search on a mono dimensional function

Changing the landscape this way does not ensure the localization of the global optimum in a very simple way. The use of an efficient global search strategy to complete the search is necessary, especially in some problems, where the objective function has a number of interesting local optima. We can remark from Figure 1, that the use of the modified objective function for the global optimization make this operation easier than the use of the original one. The local search in this work is used to reallocate the center of the circular design discussed in the following section.

3. Circular design

The *Circular Design* is a new technique that permits to generate a complete population around a central point expected to be the global optimum located between two parents X^t_{p1} and X^t_{p2}. The new population has a property that the point's distribution density decreases when we move away from the expected global optimum.

For an n dimensional problem and at the iteration t, a population of q individuals can be generated from two individuals X^t_{p1} and X^t_{p2} supposed to be the parents.

This population is located inside a hypersphere centered on:

$$X^t_c = [x^t_{c1}, x^t_{c2}, \ldots, x^t_{cn}]$$

defined as follow:

$$X^t_c = \frac{1}{2}(X^t_{p1} + X^2_{p2}) \,. \tag{1}$$

The coordinates of each individual $X^t_k = [x^t_{k1}, x^t_{k2}, \ldots, x^t_{kn}]$ $(k = 1, \ldots, q)$, are calculated using the equation system (2).

$$\begin{aligned} x^t_{k1} &= x^t_{c1} + \frac{R}{2 \cdot \pi} \cdot \theta_1 \left[\prod_{l=2}^{n} \cos \theta_l \right] \\ x^t_{kn} &= x^t_{cn} + \frac{R}{2 \cdot \pi} \cdot \theta_1 \cdot \sin \theta_2 \end{aligned} \tag{2}$$

and

$$x^t_{kr} = x^t_{cr} + \frac{R}{2 \cdot \pi} \cdot \theta_1 \left[\prod_{l=2}^{n-r+1} \cos \theta_t \right] \cdot \sin \theta_{n-r+2}$$

for $r = 2, \ldots, (n-1)$ and $k = 1, \ldots, q$, $\theta_l = \frac{2 \cdot \pi}{q} ud(j,l)$, $l = 1, \ldots, n$ represent the circular transformation of $ud = [ud_{j,i}]_{q^*n}$, a matrix of points uniformly scattered obtained by applying over the considered research space, the linear uniform design technique discussed in [9], [8]. And it has the following form:

$$ud_{ij} = (i\rho^{j-1} \text{mod } q), \qquad 1 \leq i \leq n,\ 1 \leq j \leq q\,.$$

ρ is an experimental parameter, which takes different values depending on the problem in use [9]. In this work we used the vales given in Table 1.

R, the radius of the hypersphere is given by equation (3).

$$R = \sqrt{(X^t_{p1} - X^t_{p2})^T (X^t_{p1} - X^t_{p2})}\,. \tag{3}$$

The selection process depends on the potential of the corresponding population.

Since the generated population using the circular design occupies a hypersphere it will be associated to the region represented by this hypersphere. And thus we can say that the circular design generates subregions of the search space using a set of individuals. At this level the circular design is able to construct an adaptive partitioning algorithm.

4. The adapiive partitioning algorithm

The Adaptive Partitioning Algorithm (*APA*) algorithm operates on the basis of partitioning the feasible space into non-uniform sub-regions generated by the Circular Design technique. The initial search space α_0, is successively refined until a predetermined level of precision is reached.

At the iteration t, the search space $\alpha(t)$ is partitioned into $\alpha_i(t)$ $(i = 1, \ldots, C^2_m)$ sub-regions, m is the number of points selected as super individuals and used as parents by the Circular Design technique in the generation of the C^2_m sub-regions.

4.1 *Potential of a sub region and decision factor*

The potential of a space can be evaluated using interval, statistical estimation techniques or fuzzy approach [7].

For the fuzzy approach, the degree of membership of any point, is measured by the membership function $\mu_{i,k}$ of the function to minimize $f(x)$, $x \in \alpha_i(t)$.

For the evaluation of $\mu_{i,k}$, several formulations are possible [7], we can mention the S-membership function, the Gaussian membership function or the linear membership function.

In this paper, we have chosen to compute $\mu_{i,k}$, using the modified linear membership function, described in equation (4).

$$\mu_{i,k} = (f(x_{i,k}) - f(x^*))/R_t\,. \tag{4}$$

Where $f(x^*)$ represents the smallest value of f found over all the subspaces and R_t is the range of all functions values; it is equal to difference between the greatest and the smallest value of f. This formulation allows differentiating two subspaces having very small values. In the original formulation, when this term is very small, the membership function is approximated to 1.

This information is transformed into potential value using equation (5).

$$r_k = \left[\frac{1}{q}\right] \sum_{k=1}^{q} \mu_{i,k} \cdot \exp(1 - \mu_{i,k}) \tag{5}$$

q represents the size of the sample set of the sub-region.

The issue of deciding which sub space may contain the global minimum depends on the potential of the subspace. Its value depends on the position of the region with regard to the global minimum. The best sub-region will correspond to the smallest potential.

At each iteration t, the algorithm should generate C_m^2 populations from m points chosen among the super individuals of the considered search space. Each population is defined by its own parameter vector $X^t_{i,j=1,\ldots,q}$, where q is the population size and $i = 1, \ldots, C_m^2$.

Calculating the potential of each region and deciding which subregion $\alpha_i(t)$ will be selected for the next repartitioning completes the full description of this APA.

The proposed algorithm allows us to determine the region that is most susceptible to contain the global optimum. So the described procedure is repeated till we end up with the neighborhood of the global optimum.

The circular design technique forces the sub region whose central point located at the nearest position to the global optimum to have the smallest decision factor value by ensuring a well point distribution around this optimum.

Following this procedure we ensure that the probability of missing the global optimum when t the time iteration increases goes to zero.

The local search is invoked at each generation using a simple gradient algorithm for only one time. The local search will reallocate the center of the population generated using the circular design to ensure best decision making in the selection of the subregion to be partitioned in the next iteration time t. The reallocation's effect helps when the set of super individuals took a very narrow landscape. On the other hand the employment of the local search with only the set of super individuals reduces the operation time task, and we are not obliged to change the full landscape. Thus we end up with a low cost global optimization operation.

The proposed algorithm can be summarized as follows:

(1) Scatter the initial search space using linear uniform design.

(2) Over the search space, select m super individual based on the point's functional values.

(3) Using the Circular Design technique, generate for each pair of the m points, a population of nj individuals. Each population will represent a sub-region.

(4) Apply the local search taking the center of each population as the initial point.

(5) For every population of the C_m^2 new populations evaluate the potential and the decision factor and select the generation that has the smallest decision factor.

(6) If the stop condition is not satisfied return to 1.

End

5. Numerical application

To verify the effectiveness of the proposed technique, the following test functions are used.

The obtained results are compared with those obtained using Standard Genetic Algorithm to prove that the proposed idea represents a new global search strategy. However, we are trying to show that the new technique gives better results than genetic algorithms.

The first test function is taken from [10] as a bench mark problem:

$$f(x_i)|_{i=1,\ldots,n} = k_3\Big\{ \sin^2(\pi k_4 x_1) + \sum_{i=1}^{n-1}(x_i - k_5)^2[1 + k_6 \sin^2(\pi k_4 x_{i+1})] + (x_n - k_5)^2[1 + k_6 \sin^2(\pi k_7 x_n)]\Big\}.$$

Where $k_3 = 0.1$, $k_4 = 3$, $k_5 = 1$, $k_6 = 1$, $k_7 = 0.1$ and $n = 5$.

RASTRIGIN

$$f(x_i)_{i=1,\ldots,7} = 200 + \sum_{i=1}^{7}[x_i^3 - 10 \cdot \cos(2 \cdot \pi \cdot x_i)]$$

E.KEARFOTT

$$f(x_i) = (x_1^2 - x_2)^2 + (x_2^2 - x_3)^2 + (x_3^2 - x_4)^2 + (x_4^2 - x_1)^2$$

For the first test function an initial search space $\alpha_0 = \{-5 < x_i < 5\}$ is used, the results shown in Figure 2 represent the mean functional values of both used techniques at each iteration time t, and if we compare the obtained results with those given in [13] we find that the proposed technique is more efficient than the improved genetic algorithm used in this reference.

The employments of the Rastrigin function with the New Algorithm and a Standard Genetic Algorithm (*SGA*) with two sites of crossover gives the results shown in Figure 3.

The standard genetic algorithm uses binary coding with a genome length of 10 bits of each variable, the crossover and mutation probabilities are as indicated in Table 2.

Table 1
The value of the parameters n, q and p for each problem

	q	n	p
1st problem	31	5	20
2nd problem	31	4	22
1st problem	31	7	24

Table 2
Corresponding crossover and mutation probabilities used by the SGA

	Crossover	Mutation
1st function	0.75	0.05
Rastrigin (7 variables)	0.75	0.02
E. Kearfott	0.65	0.05

The initial search space of the second test problem is given by:

$$\alpha_0 = -10 < x_i < 10, \qquad i = 1, \ldots, 7\,.$$

The mean functional values of the total population at each iteration number t are shown in Figure 2. From this result we can estimate each algorithm speed and precision. The global optimum of Rastrigin function is located at $x_i^* = 0$, for $i = 1, \ldots, 7$, with $f(x_i^*) = 130$. The centers of Circular Design at each iteration number t and their corresponding radiuses are given in Table 3. It is clear that after certain iteration numbers; the center of the CD will represent the GO. The new algorithm finds the point:

$$x^* = (0.0490\ \ 0.0499\ \ 0.0132\ \ 0.0741\ \ 0.2573\ \ 0.7569\ \ 0.2279) \cdot 10^{-9},$$

after the sixth generation with $f(x^*) = 130$.

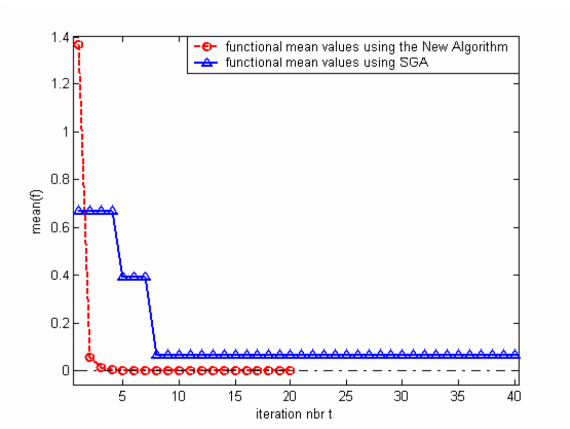

Figure 2
The use of the genetic algorithm and the new technique on the first test function

Table 3

The center of the *Circular Design* and its Radius at each iteration number t for the *Rastrigin problem*

Search region $\alpha_i(t)$								
Circular design center							Radius	t
0.1595	0.0083	0.1562	-0.0025	0.3105	0.0287	0.6576	0.65761	1
0.0009	0.0009	0.0012	-0.0013	-0.0006	0.0020	0.0036	0.00362	2
(0.0972	0.0059	0.0098	0.0073	0.0014	0.0769	$0.1101)\times 10^{-4}$	1.101×10^{-5}	3
(0.0490	0.0499	0.0132	0.0741	0.2573	0.7569	$0.2279)\times 10^{-9}$	5×10^{-10}	6

On the other hand the best minimizer that is found using the SGA after 200 generation is:

$$x^* = (-0.0098\ \ 0.0098\ \ -0.0098\ \ 0.0098\ \ -0.9286\ \ 0.0098\ \ -0.0098),$$

and the corresponding functional value is 132.562, we must keep in mind that the SGA has a population size that is approximately twice the population size of the new algorithm.

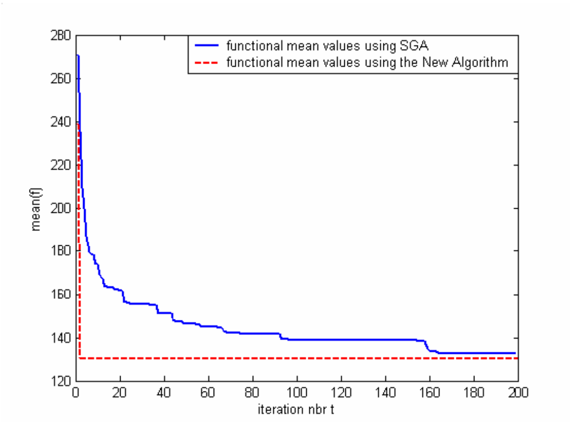

Figure 3

The use of the genetic algorithms and the new technique on the Rastigin function of seven variables

For the E. Kearfott function we started with the same initial search space as for the Rastrigin function. The results are given in Figure 4.

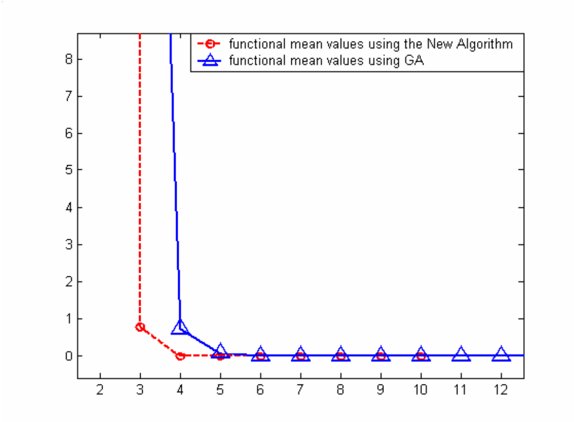

Figure 4

The use of the genetic algorithms and the new technique on the Extended Kearfott function

4. Conclusion

In this work we presented a new technique called Circular Design that is used with a local search to construct a final efficient algorithm for the global optimization of non convex functions.

The results given in this work show how this technique proceeds toward the global optimum in a minimum run time. This algorithm is applied with different functions. At the same time a parallel execution of a simple genetic algorithm to compare their results with those given in this frame work, since the genetic algorithms prove their efficiency in Large Scale Optimization.

Finally, we can say that the technique developed in this work is a satisfactory one in terms of the time they need to reach the global optimum.

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