

Using signed distance and order statistics method for fuzzy evaluation of service quality

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Abstract

Using signed distance for defuzzification and order statistics for measuring relative weight are applied in fuzzy evaluation of service quality. Many literatures reveal that service quality is usually evaluated by statistical method and also focused on one specific industry. The statistical method can estimate the mean of customers' satisfactory level of each evaluation factor and it can also show the priority factors of the service quality for improvement. However, the satisfactory level of evaluation dimension and aggregate evaluation results can't be achieved by using statistical method. In this study, the evaluation process is proposed to improve the drawbacks of common statistical methods. Furthermore, an illustrative example is given to verify the results and show its advantages.

Keywords : *Fuzzy evaluation, service quality, order statistics, signed distance.*

1. Introduction

Many researchers have undertaken the studies about the evaluation of service quality in many different service industries. It is found

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that most of these studies concentrate on specific industry and apply the statistical method. Though the statistical method can estimate the mean of customers' satisfactory level of each evaluation factor and can also show the priority factors of the service quality for improvement. However, the satisfactory level of evaluation dimension and aggregate evaluation results can not be achieved by common statistical method. On the other hand, different industry takes different evaluation structure, besides, while conducting sampling investigation, different questionnaire designs may have different items of relative importance. To improve the above problems, an algorithm of fuzzy evaluation of service quality based on signed distance and order statistics is developed to deal with the evaluation of service quality for any kind of service industries. It includes general transformation style between linguistic variables of relative importance and triangular fuzzy numbers, the customers use the order statistics as the scale to assess the relative importance and service level. In the meantime, the aggregate evaluation results can be obtained via the computation procedure of the algorithm. In order to evaluate service quality precisely, we have to distinguish service from goods and define service quality clearly. Kotler [6] mentioned that products are concrete and can be measured easily. Services, however, are noted for the characteristics of intangible, separability, perishability, and high degree of variability. Because service quality is intangible and subjective, it is not easy to develop of objective standard for evaluating their quality. Various researches have tried to survey the service quality and achieved some good point. Chien and Tsai applied triangular fuzzy number to evaluate service quality [2]. Gronroos [3] mentioned that service quality can be classified as technical and functional. Technical quality refers to "what" and functional quality concerns "how". Sasser et al [10] define the service quality in terms of material, person and equipment. Parasuraman et al [8] claimed that service quality can be measured by an assessment which consists of service performance and the customer's expectations. They developed a conceptual model of service quality that resulted in a measurement scale, the so-called SERVQUAL. Ten items is included by SERVQUAL because of some shortcomings, Parasuraman et al [9] modify their study and reduce their items to five, which are tangibility, reliability, responsiveness, assurance and empathy.

This study is intended to achieve two purposes. The first is to

develop the algorithm of fuzzy evaluation of service quality based on order statistics for surveying service quality of any kind industry. The second is to solve the shortcoming of common statistical method that the aggregate evaluation results can not be achieved.

2. Preliminaries

The following basic definitions of fuzzy set will be used throughout the paper [13].

Definition 2.1. Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is defined as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b, \\ (c-x)/(c-b), & b \leq x \leq c, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Definition 2.2. For a given $\alpha \in [0, 1]$, let $A_\alpha = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha, x \in X\}$ be denoted as a α -cut of \tilde{A} , and $A_\alpha = [\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)]$, where $\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)$ are the left-point and right-point of interval A_α , respectively.

Definition 2.3. Let $A_L(\alpha) > 0, \forall \alpha \in [0, 1]$, then \tilde{A} is denoted as a positive triangular fuzzy number (PTFN).

Definition 2.4 (see [5]). Let $\tilde{A} = (a, b, c), \tilde{B} = (p, q, r)$, for real value $e \in R, \tilde{A}, \tilde{B}$ are triangular fuzzy numbers and let $a < b < c, p < q < r$. Then, we have the following operations of triangular fuzzy number of \tilde{A}, \tilde{B} .

$$\tilde{A}(+) \tilde{B} = (a+p, b+q, c+r), \quad (2)$$

$$\tilde{A}(-) \tilde{B} = (a-r, b-q, c-p), \quad (3)$$

$$\tilde{A}(\bullet)e = (ae, be, ce), \quad e > 0. \quad (4)$$

Definition 2.5 (see [11]). Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number and $d(\tilde{A})$ be the value of defuzzification of \tilde{A} by signed distance, defined by

$$d(\tilde{A}) \equiv \frac{1}{2} \int_0^1 (\tilde{A}_L(\alpha) + \tilde{A}_R(\alpha)) d\alpha = \frac{1}{4}(a+2b+c), \quad (5)$$

where $\tilde{A}_L(\alpha) = a + (b-a)\alpha, \tilde{A}_R(\alpha) = c - (c-b)\alpha, \alpha \in [0, 1]$.

Definition 2.6 (see [1]). The order statistics of a random sample X_1, \dots, X_n are the sample values placed in ascending order. They are denoted by $X_{(1)}, \dots, X_{(n)}$.

Proposition 2.1 (see [1]). *The order statistics are random variables that satisfy $X_{(1)} \leq \dots \leq X_{(n)}$. In particular, $X_{(1)} = \min_{1 \leq i \leq n} X_i$, $X_{(2)}$ = second smallest X_i , \dots , $X_{(n)} = \max_{1 \leq i \leq n} X_i$.*

3. Methodology

3.1 Evaluation factors

In order to establish the general evaluation criteria, n factors are considered and each factors is denoted as e_j , $j = 1, 2, \dots, n$.

3.2 The fuzzy evaluation procedure

In the following, we present the procedure of fuzzy approach for evaluating the service quality based on order statistics.

- (1) *Determining the evaluation factors:* The evaluation factors are assumed having n factors.
- (2) *Transforming the linguistic variables into triangular fuzzy numbers:* According to Yu and Yao [11], the v linguistic variables can be transformed into v triangular fuzzy numbers by the following equations. The first linguistic variable L_1 can be represented as triangular fuzzy number $\tilde{L}_1 = \left(0, 0, \frac{1}{v-1}\right)$, the last linguistic variable L_v can be transformed into triangular fuzzy number $\tilde{L}_v = \left(\frac{v-2}{v-1}, 1, 1\right)$, and the rest of linguistic variables can be respectively transformed into triangular fuzzy numbers $\tilde{L}_q = \left(\frac{q-2}{v-1}, \frac{q-1}{v-1}, \frac{q}{v-1}\right)$, $q = 2, 3, \dots, v-1$. The shapes of their membership functions are listed in Figure 1. For example, if we adopt the fifth grade of linguistic variables that include very low (VL), low (L), medium (M), high (H), and very high (VH), then, these linguistic variables are transformed into triangular fuzzy numbers (illustrated in Figure 2), and the numeric results are shown in Table 1.

Table 1
Linguistic variables correspond to triangular fuzzy numbers

Linguistic variable	Triangular fuzzy number
Very Low (VL)	(0, 0, 0.25)
Low (L)	(0, 0.25, 0.5)
Medium (M)	(0.25, 0.5, 0.75)
High (H)	(0.5, 0.75, 1)
Very High (VH)	(0.75, 1, 1)

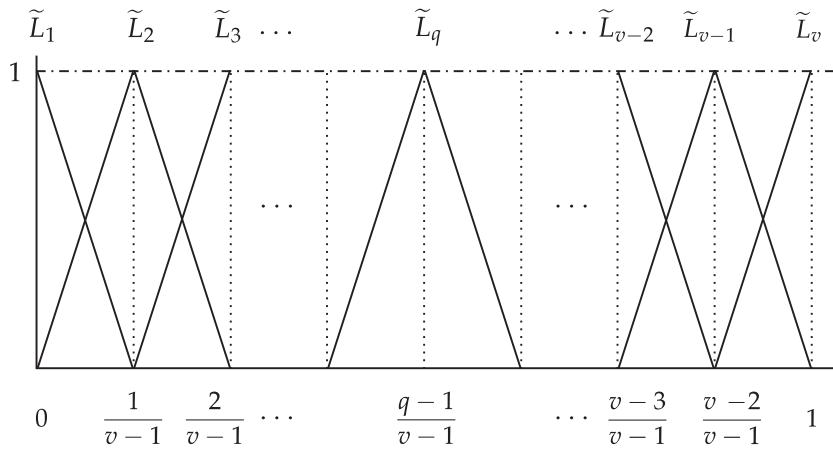


Figure 1
Fuzzy numbers of $\tilde{L}_1, \tilde{L}_2, \dots, \tilde{L}_v$

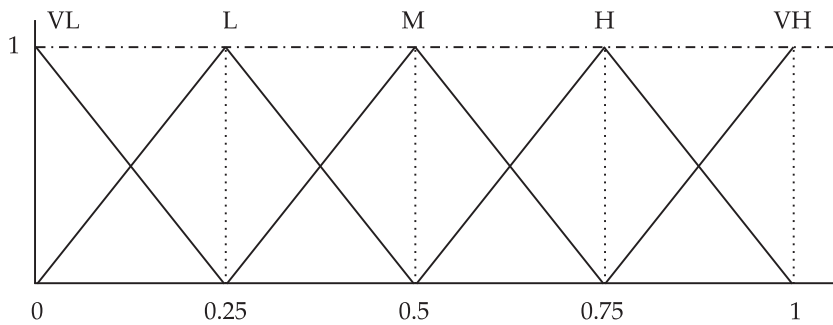


Figure 2
Fuzzy numbers of VL, L, M, H and VH

(3) *Establishing relative weight of service quality of evaluation factor:* For measuring relative importance of each evaluation factor, m customers are sampled randomly and requested to rate the relative importance of each factor in their mind. These customers use the number of $1, 2, \dots, n$, respectively to show the order of relative importance of each factor; where as 1 expresses most importance, 2 expresses second importance, and n represents the most unimportance, etc. The evaluation results is shown in Table 2. In Table 2, the k_{ij} , $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, represents the value of order of relative importance which is made by i th customer to assess j th factor; m represents the number of customers (samples). The value of order of relative importance can be replaced by evaluation value of relative importance via equation (6).

$$x_{ij} = n - k_{ij} + 1, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n \quad (6)$$

where x_{ij} represents the evaluation value of relative importance.

Table 2
The assessment results of relative importance of all evaluation factors based on data from m customers

Evaluation factor (e_i) →	e_1	e_2	⋯	e_{10}
Customer (i) ↓				
1	k_{11}	k_{12}	⋯	k_{1n}
2	k_{21}	k_{22}	⋯	k_{2n}
⋮	⋮	⋮	⋮	⋮
m	k_{m1}	k_{m2}	⋯	k_{mn}

Referring Figure 1 and Table 2, the x_{ij} can be transformed into triangular fuzzy numbers. Let $\tilde{H}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ be denoted the triangular fuzzy number of relative importance by the j th customer for rating the i th evaluation factor (x_{ij}). By taking average of each customer, we obtain \tilde{H}_j which is the triangular fuzzy numbers of average relative importance of e_j .

$$\tilde{H}_j = \frac{1}{m}(\bullet) \left(\sum_{i=1}^m \tilde{H}_{ij} \right) = \frac{1}{m}(\bullet) \left(\sum_{i=1}^m a_{ij}, \sum_{i=1}^m b_{ij}, \sum_{i=1}^m c_{ij} \right). \quad (7)$$

Through defuzzification by signed distance method (Eq. (5)), we get

H_j , the value of relative importance of each evaluation factor in the fuzzy sense, which is given by

$$H_j \equiv d(\tilde{H}_j) = \frac{1}{4m} (\bullet) \left(\sum_{i=1}^m a_{ij} + 2 \sum_{i=1}^m b_{ij} + \sum_{i=1}^m c_{ij} \right). \quad (8)$$

Finally, applying equation (9), the relative weight of each evaluation factor in the fuzzy sense (w_i) is obtained.

$$w_j = \frac{H_j}{\sum_{j=1}^n H_j}. \quad (9)$$

- (4) *Service quality assessment*: Service quality should be decided by customers, neither by manufacturers nor by sellers [6]. Since customers' points of view are the main part comparing to others, so we need to collect the relevant data through questionnaire survey. Suppose m respondents who are randomly selected are asked to assess the service quality of a specific industrial firm. These respondents rate the score (y_{ij}) of their experienced quality levels, and the score is ranged from 1 to 100 points, where y_{ij} is the score of satisfactory level of i th customer for evaluating j th evaluation factor. Based on the collected data, each average score (sample mean) and standard deviation of data of evaluation factors are calculated by using the following equations (10) and (11), respectively.

$$\bar{y}_j = \frac{1}{m} \sum_{i=1}^m y_{ij}, \quad (10)$$

$$s_j = \sqrt{\frac{1}{m-1} \sum_{i=1}^m (y_{ij} - \bar{y}_j)^2}. \quad (11)$$

Usually, it is not easy to estimate the deviation of point estimate \bar{y}_j of μ_j from the true value μ_j ; the average score of the j th evaluation factor. Therefore, instead of a point estimate, we consider $(1 - \alpha)$ confidence interval of μ_j and which is given by

$$\left[\bar{y}_j - t_{m-1}(\alpha_{j1}) \frac{s_j}{\sqrt{m}}, \bar{y}_j + t_{m-1}(\alpha_{j2}) \frac{s_j}{\sqrt{m}} \right], \quad (12)$$

where $0 < \alpha_{jk} < 1$, $k = 1, 2$, $\alpha_{j1} + \alpha_{j2} = \alpha$, s_j is given from Eq. (11), and $t_{m-1}(\alpha_{jk})$ is quantile of Student t random variable with $m - 1$

degree of freedom satisfying $P(T \geq t_{m-1}(\alpha_{jk})) = \alpha_{jk}$, $k = 1, 2$. Hence, we obtain the level $(1 - \alpha)$ fuzzy number in the following (see [4, 11]).

$$\tilde{y}_j = \left[\bar{y}_j - t_{m-1}(\alpha_{j1}) \frac{s_j}{\sqrt{m}}, \bar{y}_j + t_{m-1}(\alpha_{j2}) \frac{s_j}{\sqrt{m}} \right], \quad (13)$$

where \tilde{y}_j is triangular fuzzy number of average evaluating score of j th evaluation factor.

Since the membership grade of \tilde{y}_j is 1 under the level $(1 - \alpha)$ triangular fuzzy number and if a point estimate is move away from \bar{y}_j and close to either one of the end-points of interval of (12); i.e., $\bar{y}_j - t_{m-1}(\alpha_{j1}) \frac{s_j}{\sqrt{m}}$ or $\bar{y}_j + t_{m-1}(\alpha_{j2}) \frac{s_j}{\sqrt{m}}$, then the membership grade decreases gradually to 0. The membership grade is set to be 0, if the point estimate takes value as one of $\bar{y}_j - t_{m-1}(\alpha_{j1}) \frac{s_j}{\sqrt{m}}$ or $\bar{y}_j + t_{m-1}(\alpha_{j2}) \frac{s_j}{\sqrt{m}}$. Hence, it is reasonable to set triangular fuzzy set (12) corresponding to interval (11).

- (5) *Aggregating the evaluation point:* From Eq.(4), by multiplying every triangular fuzzy number of average evaluating score of j th evaluation factor (\tilde{y}_j) with the corresponding relative weight (w_j), we obtain the aggregate triangular fuzzy number of evaluation point of j th evaluation factor of service quality (\tilde{D}_j). The computation formula is shown as follows.

$$\begin{aligned} \tilde{D}_j &= w_j(\bullet)\tilde{y}_j \\ &= \frac{\sum_{i=1}^m x_{ij}}{\sum_{j=1}^n \sum_{i=1}^m x_{ij}} (\bullet) \left[\bar{y}_j - t_{m-1}(\alpha_{j1}) \frac{s_j}{\sqrt{m}}, \bar{y}_j, \bar{y}_j + t_{m-1}(\alpha_{j2}) \frac{s_j}{\sqrt{m}} \right] \\ &= \left[\left(\frac{H_j}{\sum_{j=1}^n H_j} \right) \left(\bar{y}_j - t_{m-1}(\alpha_{j1}) \frac{s_j}{\sqrt{m}} \right), \left(\frac{H_j}{\sum_{j=1}^n H_j} \right) \bar{y}_j, \right. \\ &\quad \left. \left(\frac{H_j}{\sum_{j=1}^n H_j} \right) \left(\bar{y}_j + t_{m-1}(\alpha_{j2}) \frac{s_j}{\sqrt{m}} \right) \right] \\ &\equiv [r_j, t_j, u_j], \quad j = 1, 2, \dots, n. \end{aligned} \quad (14)$$

Again, we apply the signed distance method (in Eq. (5)) for defuzzification of \tilde{D}_j , then the aggregate evaluation point of j th evaluation factor of service quality in the fuzzy sense (D_j) is given as follows (in Eq. (15)).

$$D_j \equiv d(\tilde{D}_j) = \frac{1}{4}(\bullet)(r_j + 2t_j + u_j). \quad (15)$$

Finally, the total aggregate evaluation point of service quality is obtained by the following equation (16).

$$D = \sum_{j=1}^n D_j. \quad (16)$$

4. Numerical example

Let $m = 500$, $n = 10$, then the evaluation procedure is depicted detail as follows:

- (1) *Determining the evaluation factors:* In this study, we assume that the number of evaluation factor is 10, i.e., e_1, e_2, \dots, e_{10} .
- (2) *Establishing relative weight of service quality of evaluation factor:* Assuming that 500 customers are randomly selected and asked to answer the questionnaire. The respondents are requested to use $1, 2, \dots, 10$ to express their feeling about the order of relative importance of 10 evaluation factors. The collected data are shown in Table 3.

Table 3

The assessment results of relative importance of all evaluation factors based on data from 500 customers

Evaluation factor(e_i) \rightarrow	e_1	e_2	\dots	e_{10}
Customer (i) \downarrow				
1	5	1	\dots	9
2	7	2	\dots	6
3	6	3	\dots	5
\vdots	\vdots	\vdots	\ddots	\vdots
500	1	4	\dots	6

The investigation results are expressed as follows. From Table 3, we see that the first customer thinks the e_1 is the fifth order of relative

importance; the e_2 is the first order of relative importance. Similarly, the second customer considers that the e_1 is the seventh order of relative importance and e_2 is the second order of relative importance. The rest data can be expressed as the same way. The original data can be transformed into the relative weight of each evaluation factor in the fuzzy sense via the Eq. (6)-Eq. (9). The detail results are shown in Table 4.

Table 4
The relative weight of each evaluation factor

Evaluation factor (e_i)	e_1	e_2	e_3	e_4	e_5
Relative weight (w_i)	0.0689	0.1079	0.0993	0.1087	0.0591
Evaluation factor (e_i)	e_6	e_7	e_8	e_9	e_{10}
Relative weight (w_i)	0.1038	0.0874	0.0847	0.1006	0.1795

- (3) *Service quality assessment*: 500 respondents who are randomly selected are asked to assess the service quality by rating the score (y_{ij}) of their experienced quality levels and the score is ranged from 1 to 100 points. Based on the collected data, each average score (sample mean) and standard deviation of evaluation factors are calculated respectively by statistical analysis (In Eq. (10) and Eq. (11)). The computing results are given in the column (a) and column (b) of Table 5.

Table 5
The statistical analysis results of rating the score of evaluation factor

Evaluation factor (e_i)	$\bar{y}_j(a)$	$s_i(b)$	$\tilde{y}_j(c)$
e_1	75.26	12.54	(74.2028, 75.26, 76.4148)
e_2	84.32	13.54	(83.1785, 84.32, 85.5669)
e_3	64.99	11.89	(63.9876, 64.99, 66.0849)
e_4	81.45	17.45	(79.9789, 81.45, 83.0569)
e_5	71.65	16.34	(70.2725, 71.65, 73.1547)
e_6	84.62	18.11	(83.0933, 84.62, 86.2877)
e_7	87.32	13.25	(86.2030, 87.32, 88.5402)
e_8	79.31	14.32	(78.1028, 79.31, 80.6287)
e_9	67.34	9.98	(66.4987, 67.34, 68.2590)
e_{10}	72.34	10.25	(71.4759, 72.34, 73.2839)

- (4) *Computing the triangular fuzzy number of average evaluating score:* Substituting the numerical values of the column (a) and column (b) of Table 5 into Eq. (10) and Eq. (11), (The α_{i1}, α_{i2} are decided by decision maker by their historical experience. In here, we suppose the $\alpha_{j1} = 0.03, \alpha_{j2} = 0.02$, then the t value can be computed, i.e., $t_{500-1}(0.03) = 1.88507, t_{500-1}(0.02) = 2.05913$). The triangular fuzzy number of average evaluating score of j th evaluation factor \tilde{y}_j can be obtained via Eq. (13) and is shown in column (c) of Table 5.
- (5) The aggregate triangular fuzzy number of evaluation point and the aggregate evaluation point in the fuzzy sense are generated via the Eq. (14) and Eq. (15) by employing the data of Table 4 and Table 5. The calculation results are given in the following Table 6.

Table 6

The aggregate triangular fuzzy number of evaluation point and the Evaluation factor

Evaluation factor (e_i)	\tilde{D}_i	D_j
e_1	(5.1118, 5.1846, 5.2641)	5.1863
e_2	(8.9759, 9.0991, 9.2336)	9.1019
e_3	(6.3526, 6.4521, 6.5608)	6.4544
e_4	(8.6964, 8.8563, 9.0311)	8.8600
e_5	(4.1554, 4.2369, 4.3259)	4.2388
e_6	(8.8781, 9.0412, 9.2194)	9.0449
e_7	(7.5332, 7.6308, 7.7374)	7.6331
e_8	(6.6136, 6.7158, 6.8275)	6.7182
e_9	(9.2670, 9.3842, 9.5123)	9.3869
e_{10}	(9.8486, 9.9676, 10.0977)	9.9704

- (6) The total aggregate evaluation point of service quality is obtained by adding each aggregate evaluation point in the fuzzy sense (in Eq. (16)). The result is 76.5948, i.e., $D = 76.5948$.

5. Conclusion

Service quality is not easy to assess precisely and the usual statistical method can not aggregate the assessment results of all evaluation factors by using different weights in different factors. In this study, we employ

a fuzzy approach for evaluating the level of service quality. To do so, respondents are asked to use order statistics to answer questionnaires for assessing the relative importance and to use 1 to 100 point to rate the service level. Then, the triangular fuzzy number is introduced to deal with the problem of point estimate and interval estimate. The aggregate evaluation results are obtained via defuzzification and multiplication of evaluation value with the corresponding relative weight.

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