

**Monitoring and incentives in a supply chain: an agency-theoretic perspective**

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**Abstract**

Monitoring and incentives in a supply chain: a agency-theoretic perspective Abstract: A principal-agent relationship can be established up for a supply chain, in which the supplier acts as the principal while the manufacturer acts as the agent. This study uses the principal-agent model to explore the relationships between multiple suppliers and a single manufacturer. A mechanism of incentives is introduced in the model, which encourage the manufacturer to make more effort to reach the expectations of suppliers; thus, they maximize profits in the supply chain. Comparisons and analyses have been made on both united- and separate-designed compensation mechanisms of multiple suppliers (principals) to a single

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manufacturer (agent) and on both multi- and single-incentive policies in this study. Fruitful results were found that will be helpful for building the partnership between supplier and manufacturer.

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*Keywords* : Supply chain, principal-agent model, multi-principal, multi-incentive mechanism, compensation plan, information asymmetric.

## 1. Introduction

In general, the supply chain is composed of many independent enterprises that have different goals and play different functions in the chain [12]. The management of any supply chain is a network that control and coordinates the functions of purchasing, producing and selling. This process will involve the supplier, producer, selling center depository and so on. Information sharing and collaboration among the members of supply chain will increase the whole supply chain's performance [20]. Any change by one of them will influence the efficiency of the supply chain [10].

The impossibility that each member of the supply chain has the same goal is implied, but it is possible for them to work together for cooperatively [13, 14]. So it does no harm to the supply chain if the chain is lightly disharmonious. Only when the efficiency of the supply chain is reduced significantly or the cost of incentive is greater than the profit incentive, would it be necessary to harmonize the members' behavior. The management of the supply chain means a method which integrates suppliers, producers, depositories and stores together effectively so as to make sure the quantity of products will be correct and the goods will be delivered at right time to the right place [3, 21]. Thus, effective management of the supply chain can meet the requirements of the service level, as well as reduce system costs and improve system performance.

As matter of fact, through information and resource integration in the supply chain, costs will be reduced and the service quality will be enhanced. However, the supply chain is a dynamic system; in addition, the members have different goals; therefore, it is hard to integrate the supply chain. The situation of competition and cooperation between members of the supply chain is similar to that found in game theory. Research on game

theory in the chain supply has received more and more attention from scholars in recent years. The numerous papers that deal with the same topic can be roughly divided into two kinds.

The first kind is based on the game theory and examines the cooperating-competing phenomenon. For example, the paper of Bylka [1], discusses competitive and cooperative policies of the vendor-buyer system. The paper supposes that the suppliers provide the batch products to the retailers under deterministic conditions, under this condition; he discussed many conditional games of cooperating-competing to find out the policy of Nash equilibrium. In addition, in Li's paper [9], discusses the improvement of inventory systems and established two models based on game theory. One is under the condition of non-cooperation; the other is under the condition of cooperation. The results showed that the profits gained with cooperation are greater than these from non-cooperation. The optimal order quantity of the retailer is larger with a system of non-cooperation, so as the price to the retailer. Hsiao and Lin [6] discuss a EOQ Model on Stackelberg Game for a supply chain, that is: the supply system contains one supplier and a single retailer, and the supplier in the channel holds monopolistic status, in which he not only has cost information about the retailer, but also has decision-making rights with regard to the lead-time. Under these circumstances, the optimal lead-time and order cycle time of the supplier and the retailer respectively are investigated, and approximate solutions for them are derived. As a result, if the supplier makes some concessions then both sides can win.

The second group of the papers are based on information economics and explore the cooperating-competing phenomenon under information symmetry and asymmetry. For example, in the paper of Lau and Lau [8], they use the two-echelon newsboy problem to discuss the interactive decisions between the supplier and the retailer. This paper is special because it supposes that information is asymmetrical. In the paper, the retailer has more market information than the supplier. It discusses many decision models on the basis of the game theory. And it concludes that enhancing the market knowledge of the retailer will definitely benefit the supplier and the whole supply chain. But enhancing the market knowledge of the supplier will only benefit the supplier. Raghunathan [11] analyzes the value of the information sharing between the supplier and the retailer by means of game theory.

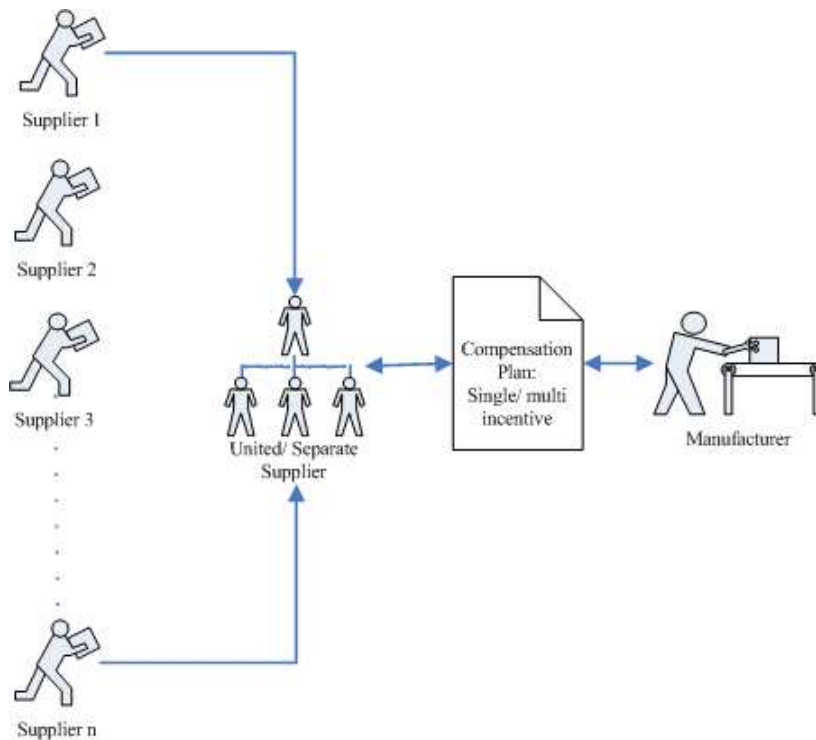
Lee and Wang [4] summarize the problems of the cooperation among the supply chain members and propose solving the cooperating-competing relationship by using the principal-agent theory. With regard to the problem of the influence of the principal on the agent, most previous papers are concerned with designing the incentive contract between the principal and the agent [2]. In general, one agent and multiple principals are more common. For example, a manufacturer (agent) has many suppliers (principals). If the efforts of the agent can be observed by the principals, it is called "information symmetry", if not, it is "information asymmetry".

In information economics, the principal-agent relation refers to any asymmetrical information exchange. The party with an information advantage is the agent, while the other is the principal. In a supply chain, the supplier will require the manufacturer to improve the quality of the supply activity and the degree of the information sharing. The manufacturer, because it has more market information, is the agent and the supplier is called the principal. In the principal-agent theory, incentive is widely used and discussed to examine how suppliers, under condition of information asymmetry, influence manufacturer's efforts to work for them. Incentive refers to a party  $A$  who makes a party  $B$ , generally party  $B$  having the private information, be favorable to itself (party  $A$ ) and at the same time the party  $B$  get its reward from  $A$ . Similarly, in a supply chain, the manufacturer (the agent) will have certain knowledge about the suppliers (the principals). The full information about the supplier is called information symmetry, which the supplier could use directly control the manufacturer. On the other hand, if the information is asymmetrical, then the supplier could use incentives to influence the manufacturer's efforts.

This paper attempts to examine the incentive problem under the conditions of "information symmetry" and "information asymmetry". Based on the principal-agent theory of game theory, by which we explore how the enterprises (the supplier) prompt the other enterprises (the manufacturers) in the supply chain. The frameworks of study are as follows: Firstly, we will use the principal-agent model to explore the cooperation-competition relationship and the effectiveness of incentive in a supply chain. Secondly, the single and multiple incentive principal-agent models will be used as an example to consider the situations of many suppliers of uniting and of separating. Finally, a comparison and summary will be made to draw out and state conclusions.

## 2. Basic models

Suppose a two-echelon supply chain that includes many suppliers and one manufacturer, with the products of the suppliers assembled and distributed to the market by the manufacturer. Owing to the fact that the suppliers can not observe the efforts made by the manufacturer, a compensation plan (incentive contract) is designed to encourage the manufacturer to make more efforts so as to increase the revenue of the whole supply chain (see Figure 1). According to agency theory, the player with the most private information is considered the agent. In this paper, since the manufacturer has more market information and information advantages, the manufacturer is regarded as the agent, with to the suppliers as principals.



**Figure 1**  
Compensation plans in a two echelon supply chain

Suppose the effort made by the manufacturer is an  $m$ -dimension  $t$ . We let vector  $x$  be a manufacturer that generates wealth for the suppliers

by expending care across  $m$  effort dimensions, with  $x$  as the output of manufacturer, that is:

$$x = t + \varepsilon. \quad (1)$$

Where  $\varepsilon$  is an unpredictable effect on the manufacturer's activities and volatility in the industry as a whole. As in Lal and Srinivasan [15], the random shock is used to represent stochastic elements in the environment and follows the normal distribution where the mean zero and diagonal variance matrix is  $\nu$ .

Suppose the number of suppliers is  $n$ , and they get profit from  $x$ . All the suppliers can observe the same vector  $x$ . We assume that their risk is neutral, and then their revenue function is linear.

Let  $b^j x$  is the supplier  $j$ 's revenue function. Superscript  $j$  is the supplier; tab above  $j$  is the transpose of vector. And let  $b$  is the sum of all  $b^j$  s; thus all suppliers' total profit is  $b'x$ , here, suppose  $b \gg 0$ , that is the supplier will not suffer a loss for the output of manufacturers.

In order to encourage the manufacturers (agents) to make the best efforts, an incentive contract designed by the suppliers (principals) should meet the two following requirements: Firstly, under the incentive mechanism, each enterprise believes that the others will choose behavior optimal to the supply chain; Secondly, the incentive must guarantee that the profit of each enterprise will grow or at least stay the same.

Now let us suppose the contract is as the follows [7]:

$$S(x) = \alpha x + \beta. \quad (2)$$

It is convenient to think of  $S$  as monetary or other rewards. The suppliers could pay rewards to the manufacturers according to the observable output  $x$  — that would be the income of the manufacturers. Through various kinds of rewards, for example, money, a certificate of award, the supplier could promote the utility of the manufacturer.

As in Lal and Srinivason [15], we suppose that the supplier's utility for income,  $S$ , increases at a decreasing rate and follows the property of constant absolute risk aversion ( $\rho$ ) over all income levels. We also assume that the exponential utility function of the manufacturer is [15]

$$u(w) = -\exp(-\rho w). \quad (3)$$

Where,  $w$  is the monetary income minus the quadratic form of the cost of expending effort  $\left(S - \frac{1}{2}t' Ct\right)$  (Note that such a functional form is also consistent with other agency-theoretic research [5, 17, 18]), and  $\rho$  is a measure of the risk aversion of the manufacturer ( $1 \geq \rho \geq 0$ ).

Suppose matrix  $C$  is a positive definite matrix, so the marginal cost of a specific effort will increase with the other effort. This implies that each suppliers could benefit in all dimensions by the efforts of the manufacturer, even if he has no direct interest in the outcome of those dimensions. So increasing one type of effort will lead to this type being substituted. For the benefit of the reading, let's positive definite matrix  $\Lambda$  as the inverse of matrix  $C$ , so elements on the diagonal are positive. If  $m = 2$ , it is easy to prove the elements not on the diagonal are negative. If  $m > 2$ , then these elements could be positive, but in most cases, they will be negative. This resembles the relationship between complements and substitutes in quantity and price in consumer-choice theory.

More discussions of the incentive problem under the "information symmetry" and "information asymmetry" will be discussed as below.

*"Information symmetry"*

If all suppliers can supervise the manufacturer directly, they can sign a contract which promises that the reward to manufacturer  $S$  is decided by the effort of manufacturer  $t$ , and the expected reward of the supplier will be

$$E[b'(t + \varepsilon)] - S = b't - S. \quad (4)$$

The utility function of the manufacturer is:

$$u(w) = -\exp(-\rho w) = -\exp\left\{-\rho\left(S - \frac{1}{2}t' Ct\right)\right\}. \quad (5)$$

The manufacturer must maximize net income  $\left(S - \frac{1}{2}t' Ct\right)$ ; because it is calculated by income minus effort cost, so it is the income-equivalence of manufacturer utility. The role of  $S$  is to transfer the income among parties, for example, promising enough utility to the manufacturer, making the manufacturer amenable to being involved in this activity. The manufacturer can make choices which can optimize profit for both parties; that is,  $t$  chosen by the manufacturer can maximize the sum of

the suppliers' profit and manufacturer's equivalence income; or maximize the total balance  $\left(b't - \frac{1}{2}t'Ct\right)$  under the first order condition of  $b = Ct$ .

Then

$$t = \Lambda b. \quad (6)$$

In most cases, the information between the manufacturer and the supplier is asymmetric; the efforts made by the manufacturer can not be observed directly by the supplier. To further explore the model, we will first examine united suppliers and separate suppliers under conditions of information asymmetry in the following two subsections.

2.1 *"Information asymmetry" — the situation of united suppliers when efforts made by the manufacturer are unobservable*

Suppose efforts by the manufacturer are not observable, and then the incentive mechanism must be based on observable output  $x$ . Also, suppose that all suppliers can share their information together completely, acting as a single entity. We specify that the manufacturer is offered a fixed return plus commission contract. Since the output is observable, commission pay is contracted on output. We assume that the amount of commission pay is linear in output. We denote fixed return by  $\beta$  and the commission rate by  $\alpha$ . The total income of manufacturer,  $S$ , may now be written as  $S = \alpha'x + \beta$ .

When they sign a contract,  $x$  is an output, payment to manufacturer is  $\alpha'x + \beta$  and the effort of the manufacturer is  $t$ , then the expected utility of the manufacturer is

$$\begin{aligned} u(w) &= -\exp(-\rho w) = -\exp\left\{-\rho\left(\alpha'x + \beta - \frac{1}{2}t'Ct\right)\right\} \\ &= -\exp\left\{-\rho\alpha't + \frac{1}{2}\rho^2\alpha'\nu\alpha - \rho\beta + \frac{1}{2}\rho t'Ct\right\}. \end{aligned} \quad (7)$$

The above equation can be:

$$\exp(-\rho y) \quad (8)$$

where

$$y = \alpha't + \frac{1}{2}\rho\alpha'\nu\alpha - \beta + \frac{1}{2}t'Ct.$$

This is the certainty equivalence income of the manufacturer, derived

by recognizing that the certainty equivalent in the case of an exponential utility function [16]. What the manufacturer should do is to maximize this certainty equivalence income. The first order condition is:

$$\alpha - Ct = 0$$

or

$$t = \Lambda\alpha. \quad (9)$$

Because the elements on the diagonal in matrix  $\Lambda$  are positive, and those not on the diagonal are negative, so any increase in value of  $\alpha$  will lead the manufacturer to increase investment in this value while reducing the benefit of other efforts at the same time.

Substitute  $t = \Lambda\alpha$  into equation  $y$ , then the certainty equivalence income of the manufacturer is

$$y = \frac{1}{2}\alpha'\Lambda\alpha - \frac{1}{2}\rho\alpha'\nu\alpha + \beta. \quad (10)$$

The expected income of the united suppliers is:

$$E[b'x - \alpha'x - \beta] = (b - \alpha)'t - \beta = (b - \alpha)'\Lambda\alpha - \beta \quad (11)$$

the optimal policy of the suppliers is to choose the value of  $\alpha$  so as to maximize the sum of equation (10) and (11), so the first order condition of  $\alpha$  is

$$\Lambda b - (\Lambda + \rho\nu)\alpha = 0. \quad (12)$$

Multiply  $C$ , then get:

$$b = (I + \rho C\nu)\alpha. \quad (13)$$

Where,  $I$  is the unit matrix in  $m$  dimension:

We can compare this result with the former optimal one (information symmetry). If  $\rho = 0$ , the equation (13) will be  $b = \alpha$ . The equation (9) will be  $t = \Lambda b$ , the same as the optimal result of "information symmetry" (6). Now let  $\rho > 0$ , because  $C$  is a positive definite matrix, and all elements are positive. The elements on the diagonal of matrix  $\nu$  are positive and the others' values are 0. As long as  $t$  is not negative, that is, if the manufacturer makes any effort, then we can set  $\alpha > 0$ . We can get:

$$b - \alpha = \rho C\nu\alpha > 0. \quad (14)$$

Or  $b > \alpha$ . So the mechanism based on an observable situation can bring less marginal income to the manufacturer than the marginal contribution from effort. (i.e., the supplier's marginal benefit  $b$  is greater than the manufacturer's  $\alpha$ , because the manufacturer has information regarding principals.) In this situation, manufacturer's effort will decrease. It results in the decrease of effort by the manufacturer and thus of total income. This implies a trade-off between efficiency and risk-sharing as a transaction cost arising from a moral hazard [2].

Substituting equation (13) into (11), we can get the optimal expected income of the suppliers, and we can get optimal situation of a supply chain:

The optimal expected income of the suppliers is:

$$y_{11} = \rho\alpha'\nu\alpha - \beta. \quad (15)$$

The certainty equivalence income of the manufacturer is:

$$y_{12} = \frac{1}{2}\alpha'\Lambda\alpha - \frac{1}{2}\rho\alpha'\nu\alpha + \beta \quad (16)$$

and, the optimal income of the supply chain is:

$$y_{13} = \frac{1}{2}\alpha'\Lambda\alpha + \frac{1}{2}\rho\alpha'\nu\alpha. \quad (17)$$

## 2.2 "Information asymmetry" the situation of separate supplier when efforts made by the manufacturer are unobservable

A supply chain is composed of many individual enterprises, each of them has its own goal, and it is difficult to achieve perfect information between the members (suppliers) in supply chain. This situation results from the suppliers' unwillingness to cooperate. Thus, each supplier has its own incentive mechanism, so the manufacturer faces various incentive mechanisms. Suppose the investment of the manufacturer is not observable, so each incentive mechanism must be based on observable output  $x$ . Here we discuss the single manufacturer. If other suppliers take a linear compensation plan too, then any suppliers can reach the optimal situation by linearity.

Use  $a^j x + \beta^j$  to stand for supplier  $j$ 's incentive mechanism. Let  $\alpha'x + \beta$  is the sum of these linear incentive mechanisms. The manufacturer makes choices by the equation (9), that is  $t = \Lambda\alpha$ . Its certainty equivalence

income can be calculated by the equation (10). But we will review the relationship of each supplier to the manufacturer. So we must know the different the ways that the manufacturer treats the supplier  $j$ .

So, we define all incentive mechanisms over the manufacturer as (with supplier  $j$  excluded)

$$A^j = \sum_{k \neq j} \alpha^k, \quad B^j = \sum_{k \neq j} \beta^k. \quad (18)$$

If the supplier  $j$  is excluded, the manufacturer will choose  $t = \Lambda A^j$ , its certainty equivalence income can be got from the equation (10), that is :

$$\frac{1}{2} A^{j'} (\Lambda - \rho\nu) A^j + B^j. \quad (19)$$

If it includes supplier  $j$ , then the certainty equivalence income can be got from the equation (10), that is:

$$\alpha = A^j + \alpha^j, \quad \beta = B^j + \beta^j. \quad (20)$$

We can write the equation (10) as

$$\frac{1}{2} (A^j + \alpha^j)' (\Lambda - \rho\nu) (A^j + \alpha^j) + B^j + \beta^j.$$

So the relationship between supplier  $j$  and the manufacturer can result in extra income of the manufacturer:

$$A^{j'} (\Lambda - \rho\nu) \alpha^j + \frac{1}{2} \alpha^{j'} (\Lambda - \rho\nu) \alpha^j \beta^j. \quad (21)$$

The expected surplus of the supplier  $j$  is

$$b^j t - \alpha^{j'} t - \beta^j = (b^j - \alpha^j)' \Lambda (A^j + \alpha^j) - \beta. \quad (22)$$

If the supplier  $j$  offers a fixed income to the manufacturer regardless of what efforts the manufacturer has made, then the income of the supplier  $j$  is  $b^j \Lambda A^j$ , so the difference is:

$$b^j \Lambda \alpha^j - \alpha^{j'} \Lambda \alpha^j - \alpha^{j'} \Lambda A^j - \beta^j. \quad (23)$$

The difference between the manufacturer and the supplier  $j$  is caused by this commission contract.

The function of  $\beta^j$  is to transfer the surplus among parties. If the supplier  $j$  chooses  $\alpha^j$  this will maximize the total surplus to both sides.

That is

$$b^j \Lambda \alpha^j - \rho A^j \nu \alpha^j - \frac{1}{2} \alpha^j (\Lambda + \rho \nu) \alpha^j. \quad (24)$$

The supplier  $j$  will not cooperate with the other suppliers, and as the given  $A^j$ , it will choose  $\alpha^j$ . Its first order condition is:

$$\Lambda b^j - \rho \nu A^j - (\Lambda + \rho \nu) \alpha^j = 0. \quad (25)$$

Or, multiply  $C$  on both sides, then,

$$b^j = (I + \rho C \nu) \alpha^j + \rho C \nu A^j. \quad (26)$$

This is the first order condition under the given  $A^j$ ; in other words, this is the best action in decision-making from the whole range of supplier choices. Thus, the other suppliers will follow supplier  $j$  in making a similar decision. (i.e., suppliers must find the optimal  $\alpha^j$  maximum of equation (24) so that they will get the maximum profit for both side.) It is the supplier  $j$ 's optimal choice. This relationship is functional to all suppliers. Adding  $\alpha^j$  by  $j$ , and recognizing the sum of  $A^j = \alpha - \alpha^j$  is  $(n - 1)\alpha$ , where  $n$  is the number of the suppliers, we can get:

$$b = (I + n\rho C \nu) \alpha. \quad (27)$$

The equation (27) implies a final decision will be made by each supplier according to its full consideration of the suppliers' decision.

Comparing sections 2.2 and 2.2, in the incentive mechanism of equation (13) all suppliers are united together, while this does not happen in equation (27). These two equations (13)-(27) are very similar apart from one more " $n$ " in equation (27). If the suppliers are not cooperative, the risk aversion of the manufacturers will multiplied  $n$  times. This influence and the number of the suppliers are in direct proportion, so the power of incentive and the numbers of the suppliers are in inverse proportion. So information asymmetric is less powerful with separated rather than united suppliers.

Substituting the equation (27) into (11), we can get the optimal expected income of supplier, and the optimal situation of the supply chain is:

The optimal expected income of the suppliers is

$$y_{21} = n\rho \alpha' \nu \alpha - \beta. \quad (28)$$

The certainty equivalence income of the manufacturer is

$$y_{22} = \frac{1}{2}\alpha' \wedge \alpha - \frac{1}{2}\rho\alpha' \nu \alpha + \beta, \quad (29)$$

and, the optimal income of the supply chain is

$$y_{23} = \frac{1}{2}\alpha' \wedge \alpha + \left(n - \frac{1}{2}\right) \rho\alpha' \nu \alpha, \quad n \geq 2. \quad (30)$$

Suppliers like to encourage the manufacturer to work for the suppliers by a monetary premium; however, there is a limit on this method of incentive. According to the backward-bending labour supply curve [19], as income rises, people like to enjoy more leisure time and are less willing to work. While it is true that the manufacturer gets more commission by working harder, their marginal utility will decrease. In this situation, the suppliers may consider designing new methods of incentive, including not only monetary commission but also other kinds so as to encourage the manufacturer. In the next section, we will study the situation of multiple incentives, and the model for that will follow this section's model.

### 3. The multi-incentive compensation plan

Suppose the supplier has two ways to compensate the manufacturer: monetary premium and reputation, for example, IBM and Microsoft not only give economic rewards but also grant some of their suppliers "excellent agents". This is one type of efficient incentive, by which the manufacturer can develop more operations, thus get more profits. Suppose the effort of manufacturer  $t$  is not observable, and then the incentive mechanism must be based on the observable output  $x$ . The following two subsections will discuss the cases united and separate supplier in detail when operating under a multi-incentive compensation plan.

#### 3.1 *United suppliers with multi- incentive compensation plan*

Suppose the output of the manufacturer is  $x$  and the payment to the manufacturer is  $\alpha'_1 x + \beta$ , meanwhile the other incentive is  $\alpha'_2 x$ , where the supplier need not pay money to the manufacturer. The expected income of the supplier is

$$y = \rho\alpha'_1 \nu (\alpha_1 + \alpha_2) - \beta. \quad (31)$$

The certainty equivalence income of the manufacturer is

$$y = \alpha_1' t + \beta + \alpha_2' t - \frac{1}{2} \rho \alpha_1' \nu \alpha_1 - \frac{1}{2} \rho \alpha_2' \nu \alpha_2 - \frac{1}{2} t' C t. \quad (32)$$

Its first order condition is:

$$(\alpha_1 + \alpha_2) = C t. \quad (33)$$

That is

$$t = \Lambda(\alpha_1 + \alpha_2). \quad (34)$$

Substitute equation (34) into (32), and the optimal certainty equivalence income of the manufacturer is:

$$y = \frac{1}{2} (\alpha_1 + \alpha_2) \Lambda(\alpha_1 + \alpha_2) - \frac{1}{2} \rho \alpha_1' \nu \alpha_1 - \frac{1}{2} \rho \alpha_2' \nu \alpha_2 + \beta \quad (35)$$

and, the expected income of united suppliers is

$$E[b'x - \alpha_1'x - \beta] = (b - \alpha_1)' t - \beta = (b - \alpha_1)' \Lambda(\alpha_1 + \alpha_2) - \beta. \quad (36)$$

The optimal policy for the united suppliers is to choose  $\alpha_1$ ,  $\alpha_2$ , so as to maximize the sum of equations (35) and (36). The total certainty equivalence income of the supply chain is

$$\arg \min_{[\alpha_1, \alpha_2]} \left\{ \frac{1}{2} (\alpha_1 + \alpha_2)' \Lambda(\alpha_1 + \alpha_2) - \frac{1}{2} \rho \alpha_1' \nu \alpha_1 - \frac{1}{2} \rho \alpha_2' \nu \alpha_2 + (b - \alpha_1)' \Lambda(\alpha_1 + \alpha_2) \right\}. \quad (37)$$

The first order condition of  $\alpha_1$  is

$$\Lambda(b - \alpha_1) - \frac{\rho}{2} \nu \alpha_1 = 0. \quad (38)$$

That is

$$\left( I + \frac{\rho}{2} C \nu \right) \alpha_1 = b. \quad (39)$$

The first order condition of  $\alpha_2$  is

$$\Lambda(b + \alpha_2) - \frac{\rho}{2} \nu \alpha_2 = 0. \quad (40)$$

That is

$$\left( I - \frac{\rho}{2} C \nu \right) \alpha_2 = b. \quad (41)$$

Substituting equation (38) into (36) will get the optimal expected income

of the suppliers, and we can get the optimal situation of the supply chain:

The optimal expected income of the supplier is

$$y_{31} = \rho\alpha'_1\nu(\alpha_1 + \alpha_2) - \beta. \quad (42)$$

The certainty equivalence income of the manufacturer is

$$y_{32} = \frac{1}{2}(\alpha_1 + \alpha_2)'\Lambda(\alpha_1 + \alpha_2) - \frac{1}{2}\rho\alpha'_1\nu\alpha_1 - \frac{1}{2}\rho\alpha'_2\nu\alpha_2 + \beta. \quad (43)$$

The optimal income of the supply chain is

$$y_{33} = \frac{1}{2}(\alpha_1 + \alpha_2)'\Lambda(\alpha_1 + \alpha_2) - \frac{1}{2}\rho\alpha'_1\nu\alpha_1 + \rho\alpha'_1\nu\alpha_2 - \frac{1}{2}\rho\alpha'_2\nu\alpha_2 + \beta. \quad (44)$$

The individual supplier in a supply chain may have difficulty integrating with other members, will next discuss the situation of non-cooperation.

### 3.2 Separated suppliers with multi-incentive compensation plan

In this situation the suppliers don't cooperate with each other. Each supplier will choose two incentive mechanisms, so the manufacturer will face a lot of incentive mechanisms. Suppose the effort of the manufacturer is not observable, each incentive mechanism must be based on the observable output  $x$ .

We define the sum of the other suppliers' incentives (except supplier  $j$ ) as:

$$A_1^j x + B^j, \quad A_2^j x$$

where  $A_1^j = \sum_{k \neq j} \alpha_1^k$ ,  $B^j = \sum_{k \neq j} \beta^k$ ,  $A_2^j = \sum_{k \neq j} \alpha_2^k$ .

If the supplier  $j$  is excluded that means  $(n - 1)$  suppliers are in cooperation, then according to the equation (34), the manufacturer will choose:

$$t = \Lambda(A_1^j + A_2^j). \quad (45)$$

Here, according to the equation (35), the certainty equivalence income of the manufacturer is:

$$y_1 = \frac{1}{2}(A_1^j + A_2^j)'\Lambda(A_1^j + A_2^j) - \frac{1}{2}\rho A_1^{j'}\nu A_1^j - \frac{1}{2}\rho A_2^{j'}\nu A_2^j + B^j. \quad (46)$$

If the supplier  $j$  is included that means all suppliers are in cooperation,

the certainty equivalence income of the manufacturer is:

$$\begin{aligned} y_2 = & \frac{1}{2}(A_1^j + A_2^j + \alpha_1^j + \alpha_2^j)' \Lambda (A_1^j + A_2^j + \alpha_1^j + \alpha_2^j) \\ & - \frac{1}{2} \rho (A_1^j + \alpha_1^j)' \nu (A_1^j + \alpha_1^j) - \frac{1}{2} \rho (A_2^j + \alpha_2^j)' \nu (A_2^j + \alpha_2^j) \\ & + (B^j + \beta^j). \end{aligned} \quad (47)$$

Thus, the extra income of the manufacturer after establishing the commission contract relationship with the supplier  $j$  is  $y_2 - y_1$ , that is:

$$\begin{aligned} y_3 = & \frac{1}{2}(\alpha_1^j + \alpha_2^j)' \Lambda (\alpha_1^j + \alpha_2^j) + (A_1^j + A_2^j)' \Lambda (\alpha_1^j + \alpha_2^j) \\ & - \frac{1}{2} \rho \alpha_1^{j'} \nu \alpha_1^j - \frac{1}{2} \rho \alpha_2^{j'} \nu \alpha_2^j - \rho A_1^{j'} \nu \alpha_1^j - \rho A_2^{j'} \nu \alpha_2^j + \beta^j. \end{aligned} \quad (48)$$

The expected income of the supplier  $j$  is:

$$\begin{aligned} E[b^j x - \alpha_1^{j'} x - \beta^j] &= (b^j - \alpha_1^j) t - \beta^j \\ &= (b^j - \alpha_1^j)' \Lambda (A_1^j + \alpha_1^j + A_2^j + \alpha_2^j) - \beta^j. \end{aligned} \quad (49)$$

If there is no commission contract relation between the supplier  $j$  and the manufacturer ( $\alpha_1^j = \alpha_2^j = 0$ ), then the income of the supplier  $j$  is  $b^j \Lambda (A_1^j + A_2^j)$ , so the extra income of the supplier  $j$  after establishing the relationship with the manufacturer is:

$$\begin{aligned} y_4 = & b^j \Lambda (\alpha_1^j + \alpha_2^j) - (A_1^j + A_2^j)' \Lambda \alpha_1^j - \alpha_1^{j'} \Lambda \alpha_1^j - \alpha_1^{j'} \Lambda \alpha_2^j \\ & - \alpha_1^{j'} \Lambda \alpha_2^j - \beta^j. \end{aligned} \quad (50)$$

The optimal policy for the supplier  $j$  is to choose  $\alpha_1^j, \alpha_2^j$ , so as to maximize the sum of equations (48) and (49)  $y_3 + y_4$ .

The first order condition of  $\alpha_1^j$  is:

$$\Lambda b^j - \rho \nu A_1 - \Lambda \alpha_1^j = 0. \quad (51)$$

Add equation (48) by  $j$  (suppose the number of the suppliers is  $n$ ), then:

$$b = (I + n \rho C \nu) \alpha_1. \quad (52)$$

The first order condition of  $\alpha_2^j$  is

$$\Lambda (A_1 + A_2) - \rho \nu A_2 + \Lambda b^j - \Lambda \alpha_1^j = 0. \quad (53)$$

Add equation (50) by  $j$

$$b = (n - 1)\alpha_1 + n(I - \rho C\nu)\alpha_2. \quad (54)$$

Here,  $A_1 = \sum_{j=1}^n \alpha_1^j = \alpha_1$ ,  $A_2 = \sum_{j=1}^n \alpha_2^j = \alpha_2$ ,  $b = \sum_{j=1}^n b^j$ .

Substitute equation (54) into (36) we can get equation (55), and the optimal income situation of the supply chain:

The expected income of the supplier is:

$$y_{41} = n\rho\alpha_1'\nu(\alpha_1 + \alpha_2) - \beta. \quad (55)$$

The certainty equivalence income of the manufacturer is:

$$y_{43} = \frac{1}{2}(\alpha_1 + \alpha_2)'\Lambda(\alpha_1 + \alpha_2) - \frac{1}{2}\rho\alpha_1'\nu\alpha_1 - \frac{1}{2}\rho\alpha_2'\nu\alpha_2 + \beta, \quad (56)$$

and, the optimal income of the supply chain is

$$y_{43} = \frac{1}{2}(\alpha_1 + \alpha_2)'\Lambda(\alpha_1 + \alpha_2) - \frac{1}{2}\rho\alpha_1'\nu\alpha_1 - \frac{1}{2}\rho\alpha_2'\nu\alpha_2 + n\rho\alpha_1'\nu(\alpha_1 + \alpha_2). \quad (57)$$

We will summarize and compare the above models in the next section.

#### 4. Summary and comparison

The effect of the multi-incentive principal-agent model is better than the single-incentive one, for example, when  $n$  suppliers are united; the suppliers will get  $y_{21} - y_{31} = \rho\alpha_1'\nu\alpha_2$  more in the multi-incentive model than in the single-incentive one. The manufacturer will get  $y_{32} - y_{12} = \alpha_1'\Lambda\alpha_2(\Lambda - \rho\nu)\alpha_2$  more, and the supply chain will get  $\alpha_1(\Lambda + \rho\nu)\alpha_2 + \frac{1}{2}\alpha_2'(\Lambda - \rho\nu)\alpha_2$  more,  $(y_{32} - y_{11}) - (y_{31} - y_{11}) = \alpha_1'(\Lambda - \rho\nu)\alpha_2 + \frac{1}{2}\alpha_2'(\Lambda - \rho\nu)\alpha_2$  and the manufacturer will get more benefit than the supplier does. (Please see Table 1)

We found that, whether single or multi-incentive mechanisms are used, a separate policy is more favorable than a united policy to the suppliers. The former can greatly enhance the profit of the suppliers (e.g. for the single incentive model, change the united policy  $y_{11} = \rho\alpha_1'\nu\alpha_1 - \beta$  into the separate policy  $y_{21} = n\rho\alpha_1'\nu\alpha_1 - \beta$ ), but the profit of the manufacturer does not change. It is important that any non-cooperative supplier can potentially get more benefits, but also faces greater risk.

**Table 1**  
**The comparison of the single and multi-incentive multi-principal – agent model**

	Single incentive	Multi incentive
United suppliers $n$	<p>Suppliers: <math>y_{11} = \rho\alpha'_1\nu\alpha_1 - \beta</math></p> <p>Manufacturer: <math>y_{21} = \frac{1}{2}\alpha'_1\Lambda\alpha_1 - \frac{1}{2}\rho\alpha'_1\nu\alpha_1 + \beta</math></p> <p>Supply chain: <math>y_{13} = \frac{1}{2}\alpha'_1\Lambda\alpha_1 + \frac{1}{2}\rho\alpha'_1\nu\alpha_1</math></p>	<p>Suppliers: <math>y_{31} = \rho\alpha'_1\nu(\alpha_1 + \alpha_2) - \beta</math></p> <p>Manufacturer: <math>y_{32} = \frac{1}{2}(\alpha_1 + \alpha_2)'\Lambda(\alpha_1 + \alpha_2) - \frac{1}{2}\rho\alpha'_1\nu\alpha_1 - \frac{1}{2}\rho\alpha'_2\nu\alpha_2 + \beta</math></p> <p>Supply chain: <math>y_{33} = \frac{1}{2}(\alpha_1 + \alpha_2)'\Lambda(\alpha_1 + \alpha_2) + \frac{1}{2}\rho\alpha'_1\nu\alpha_1 + \rho\alpha'_1\nu\alpha_2 - \frac{1}{2}\rho\alpha'_2\nu\alpha_2</math></p>
Separate suppliers $n$	<p>Suppliers: <math>y_{21} = n\rho\alpha'_1\nu\alpha_1 - \beta</math></p> <p>Manufacturer: <math>y_{22} = \frac{1}{2}\alpha'_1\Lambda\alpha_1 - \frac{1}{2}\rho\alpha'_1\nu\alpha_1 + \beta</math></p> <p>Supply chain: <math>y_{23} = \frac{1}{2}\alpha'_1\Lambda\alpha_1 + n\rho\alpha'_1\nu\alpha_1 - \frac{1}{2}\rho\alpha'_1\nu\alpha_1, n \geq 2</math></p>	<p>Suppliers: <math>y_{41} = n\rho\alpha'_1\nu(\alpha_1 + \alpha_2) - \beta</math></p> <p>Manufacturer: <math>y_{42} = \frac{1}{2}(\alpha_1 + \alpha_2)'\Lambda(\alpha_1 + \alpha_2) - \frac{1}{2}\rho\alpha'_1\nu\alpha_1 - \frac{1}{2}\rho\alpha'_2\nu\alpha_2 + \beta</math></p> <p>Supply chain: <math>y_{43} = \frac{1}{2}(\alpha_1 + \alpha_2)'\Lambda(\alpha_1 + \alpha_2) - \frac{1}{2}\rho\alpha'_1\nu\alpha_1 - \frac{1}{2}\rho\alpha'_2\nu\alpha_2 + n\rho\alpha'_1\nu(\alpha_1 + \alpha_2)</math></p>

## 5. Conclusion

Based on the principal-agent model, we take the supplier as a principal and the manufacturer as an agent; in which the principal-agent model of multi-principal represent a manufacturer and  $n$  suppliers. By using this approach, suppliers should be able to design an appropriate mechanism to encourage the manufacturer to work hard and to maximize profits of the whole supply chain. Our approach considered two questions: firstly, "which is more effective, united suppliers or separate suppliers?" secondly, "on the basis of the single reward mechanism, is it more efficient for the manufacturer to adopt the multi-incentive model?"

We examined these two problems and concluded that

- (1) As an incentive to the manufacturer, the effect of the united supplier model is better than that of the separate one. When the number of the suppliers' increases, the gap between the effects of these two different models will become larger.
- (2) The multi-incentive mechanism is more favorable than the single incentive mechanism, and the manufacturer will get more benefit than the supplier.
- (3) In the supply chain, whether a single or multi-incentive mechanism is in operation, the separate policy is more favorable than the united policy to the suppliers. In addition, the former can greatly enhance the benefit of the suppliers, but the income of the manufacturer does not change.

Although we obtained the valuable results above, future study may be need to consider the effects of multi- and single-incentive under the principal-agent model of multi-principal and multi-agents in a supply chain to obtain a more complete model which is undergoing studying now.

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