

## Issues concerning block trading and transaction costs

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### Abstract

Measuring, controlling and minimising transaction and execution costs for institutional investors are becoming increasingly important. This paper discusses the issues involved and presents a theoretical model which minimises the expected cost of Block Trading. Transaction costs and Execution costs are fully defined and discussed. Unfortunately, the data necessary to analyse many questions of interest is very difficult or impossible to obtain. In reality, Execution costs are not directly observable. Therefore, many different measures exist, each with its own advantages and disadvantages. Theoretically, execution costs can be measured via price impact functions. Some of the most important issues concerning block trading is market capitalisation, volume traded, speed of execution, the inventory levels, the trader involved and the firm involved. All of the issues are discussed from a general perspective, therefore it is not discussed from the view of one particular market.

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*Keywords* : *Transaction costs, execution costs, block trading, dynamic programming.*

### 1. Introduction

Execution and transaction costs are becoming increasingly important, especially for institutional investors such as mutual and pension funds. The literature concerning transaction costs is large and diverse, see the reference list. Execution costs can have a significant impact on investment performance. For example, Perold [30] observed that a hypothetical or

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'paper' portfolio constructed according to the Value Line rankings outperforms the market almost 20% per year during the period from 1965 to 1986, whereas the actual portfolio (the Value Line Fund) outperformed the market by only 2.5% per year, the difference arising from execution costs. This paper firstly introduces Block Trades and how they are usually transacted. Most Block Trades are negotiated by Sellers rather than Buyers [32]. There are many reasons for this and impart it can be explained by the psychological observation that large buyers are less likely than large sellers to initiate transactions, especially after bad news [32]. Traders will generally take longer to execute a buy programme than the an equivalent sell programme. Traders perceive that price impacts of buys are greater than sells [22]. Section 2 discusses transaction and execution costs. In particular, these costs are defined and some factors that affect these costs are discussed. Many researchers have experimented with trading algorithms to test out different trading styles [21]. These algorithms have many simplifying assumptions. For example, [21] assumed the market consistently maintained a certain level of liquidity and an individual traders transactions did not affect the market expectations. Traders who follow a programmed trading style may get the reputation of being an uninformed trader. In some situations, market makers can profit from uninformed traders. Conversely, a trader who falsely implies that he is an uninformed trader could also profit [16]. Section 3 discusses the computation required to theoretically minimise the expected cost of trading a large block of shares over a fixed finite time horizon. This paper uses a model derived by Dimitris Bertsimas and Andrew Lo [8]. They argue that the problem is a Stochastic Dynamic Programming one. Specifically, given a fixed block of shares ( $\bar{S}$ ) to be traded within a fixed finite number of periods  $T$ , and given the execution price of each transaction is a function of the number of shares traded and the market conditions, it is then theoretically possible to obtain an optimal sequence of transactions which minimises the expected cost of trading  $\bar{S}$  shares within  $T$  periods. As discussed in the paper [8]. The analytical solution to the problem will allow sell transactions within the buy programme. It is computationally infeasible to impose a nonnegativity constraint on the analytical solution to the problem. This paper obtains results by a Discretised Dynamic Programming method and as a result this problem is overcome. The optimal solution is then converted to a dynamic optimisation via a grid search at each stage. This paper attempts to discuss the computation required for the discretised route. Bertsimas [8] showed that a naive trading strategy is only optimal

(trading equal numbers of shares over the  $T$  periods) when you assume the price impact of your trade is a function of the trade size. Once you incorporate an information component to the price impact function the naive strategy is unlikely to be the optimal policy. Section 4 presents some of the results and Section 5 offers conclusions and discusses the limitations of the findings.

## 2. Block trading

A *block trade* is transactions of a particular share (or portfolio of shares) which is larger than normal for the market given the ruling market conditions. Some specify a block as a transaction involving at least 10,000 shares [32]. Other analysts believe a block trade involves a large number of shares that if traded badly may adversely impact the trade price, thus the liquidity and the market capitalisation of the share is important. In 1986 Block trades accounted for 49% of NYSE share volume [29], by 1996 it had increased to 56% [39]. In the US, these transactions are usually negotiated in the upstairs market, where institutional sales traders and block positioners of major security houses negotiate transactions. The Paris Bourse also provides mechanisms for putting together large transactions on the floor. How block trades are transacted and the market structure is a very in depth subject. This section attempts to indicate the main issues and refer the reader to suitable references. The main issue concerning transacting a large number of shares is the liquidity of the share. A share with a low market capitalisation is expected to encounter a higher expected cost of transacting than a share with a high market capitalisation. In Ronald [31] p. 28, a block transaction of 0.29%, which represents the percentage of shares outstanding, would expect to have a cost of 1.1% for a high market capitalised stock (\$100 Billion), while for a low capitalisation stock (\$10 Billion), the expected cost is 6.2%. A highly liquid share can sometimes be efficiently executed through one market maker. Other traders will simultaneously trade the same share in a number of different markets. A trader can also post orders simultaneously to a number of different market makers in the same market. Schwartz [32] gives a detailed description of the Block Trading process in the US. Hansch [18] offers an insight into Block Trading on the London Stock Exchange. One of the key features of the London market is that market makers are prepared to quote prices in large sizes. The speed of executing buy and sell decisions have significant a impact on transaction costs [10].

### 3. Execution and transaction costs

Transaction costs can be summarised as:

$$\begin{aligned} \text{transaction costs} &= \text{commissions} + \text{fees} + \text{taxes} \\ &+ \text{execution costs} + \text{opportunity costs} \end{aligned}$$

An execution cost represents the difference between the execution price of a security and the price that would have existed in the absence of the trade.

$$\text{Execution costs} = \text{market impact costs} + \text{market timing costs}$$

The market impact cost represents the cost resulting from the bid-ask spread and the price extracted by the dealers to mitigate their risk that an investor demand for liquidity is information-motivated. Estimates of the quoted spread as a percentage of the stock price varies considerably, from less than 0.5% for the most liquid stocks to 4-6% for the least liquid stocks, [27, 19]. Bid-Ask spreads are poor indicators of the cost of transacting with a market maker. Often transactions will be transacted in between the quoted bid-ask spread. Bid and Ask prices will generally increase (decrease) during a buy (sell) order. Finally, a block transaction need not occur at the Bid or Ask prices. This is common in the upstairs markets and automatic crossing systems [23].

The market timing cost represents the cost resulting from the adverse price movement during the transaction which cannot be attributed to the particular transaction.

Execution costs are related to both the demand for liquidity and the trading activity on the trade date.

Information motivated trades are trades that take place when investors believe they possess pertinent information not currently reflected in the security price. These trades increase the market impact cost because of the speed of execution, also the market maker may believe the trade is information driven and therefore increases the bid-ask spread to provide some protection.

The problem with measuring execution costs is that the true measure is the difference between the price of the security in the absence of the trade and the execution price, which is not observable. Furthermore, the execution prices are dependent on the supply and demand conditions at the margin. This means the execution price realised by an investor is the consequence of the structure of the market mechanism, the demand

for liquidity, and the competitive forces of the investors with similar motivations for trading. Often, the price in the absence of the trade is known as the unperturbed price. The unperturbed price is conceptually simple, but in reality it is difficult to measure. Many methods exist, such as a pretrade measure, a post trade measure, a weighted average measure, a volume weighted average measure, etc. The different measures can calculate wildly different execution costs.

Execution costs and opportunity costs are the implicit costs of trading. Overall, considerable disagreement surrounds how best to measure implicit trading costs, [5, 7, 2, 14, 37]. The availability of high quality data has increased, thus the measurement of the true cost of trading is improving. An opportunity cost is associated with the cost of only partially filling an order or not executing it at all. Also, an order may be executed with a delay, during this delay the price may move in an undesirable direction. For some investors, such as passive investors, opportunity costs are zero.

There are many ways to measure execution costs [9, 7]. The closest true measure of the execution cost is the pre-trade measure [9]. In reality, this measure is subject to gaming by traders, but in a computer simulation this is not a problem. This work used a simplified version of this measure, as discussed later. A popular way of measuring execution costs involve Volume-Weighted Average Price's, again this can be gamed by traders [23]. Pretrade measures can estimate the permanent and temporary effects on the price.

Kawahara [21] discusses why the performance of a new investment scheme is usually different to what is predicted by a simulation model that uses daily closing prices or monthly closing prices. The discrepancy can occur because the simulation model fails to model the risks involved in the execution of transactions. Kawahara [21] summarised this in the following way:

$$P_T = CP + (\Delta P + \Delta E) + DL \quad (1)$$

$$P_T = (P_L + DF) + (\Delta P + \Delta E) + DL. \quad (2)$$

Where:

$P_T$  = expected price at time  $T$

$CP$  = previous days closing price

$P_L$  = last trading price

$DF$  = changes in fundamentals between the last time the issue was traded and the present time

$\Delta P$  = expected returns

$\Delta E$  = discrepancy with forecast

$DL$  = risks involved in executing the transaction (market impact)

Keim and Madhavan [25] found that commission costs are low overall, in particular, about 0.2% of the trade value. Stoll [33] reported that commissions in 1992 averaged 7.9 cents, 0.24% of the market value of the market value of the trade. Some studies report slightly higher or lower commission costs than indicated. Commission costs over time have declined. Stoll [33] reported commission costs in 1982 were 0.58% of the market value, which is twice the 1992 level. This can be explained by increased institutional trading, increased competition and technology, such as automatic crossing systems. Similar results can be obtained for most major markets.

Actions taken by investors and traders are typically made upon hypotheses about future states of a world that is itself in part the consequences of these hypotheses. Investors and traders predictions may be depend upon the predictions others might form, and the predictions of these might depend upon the predictions they believe the original group might form. In essence, an individuals action to some degree will affect the conjectures and actions of others, thus investment decisions, and in turn share prices. The larger the transaction, the larger the potential impact. Therefore, it is not just the size of the transaction that affects the price. It also includes the trader involved, the firm involved [11], the share involved, the speed of execution and the method of execution. In effect, transactions give out signals to others. If others expect the transaction their predictions about the future is reinforced. But if they did not then they may change their future behaviour. Also the original trader will be testing the assumed actions of others, thus the original trader will determine future actions given the current actions of others. Historically, the possibility of artificially influencing stock prices has been an important issue. In all stock markets throughout the ages, some traders have discovered that they can profitably manipulate stock prices [1].

Patient trading requires trading shares at a particular price, rather than at a particular time. This can be achieved in many ways, such as limit orders, or simply waiting for the right time. Some investment managers prefer to bear the cost of instant liquidity, especially if they believe the market price will move in an undesirable direction, [10,21]. There are many factors that make trading a large number of shares difficult. Given a trade is difficult, a trader can consider adjusting the investment style by considering the following. Firstly, the order size relative to average daily

volume or shares outstanding. Secondly, the trade duration or number of subtrades. Thirdly, the order type. For example, market order, limit order, a crossing order, etc. Fourthly, the investment style, whether to be a passive investor or active. Finally, in the US an investor can reduce the price impact by trading in the upstairs market. There are some factors that the investor has no control over, such as the natural illiquidity of the share, stock market volatility, market structure and the traders reputation or ability. Keim and Madhavan [23] offers a very detailed discussion of these issues.

Abel/Noser Corporation have developed an industry standard transaction measurement system in the US. They measure about one trillion dollars of pension equity transactions annually. Abel/Noser have developed a piece of software that combines market information, order control, trading strategy and trade measurement in a real time environment. The system aims to study, measure and reduce trading costs. The system also offers advice on the maximum number of shares that can be traded at any time, so the trade does not adversely affect the price.

Barra Portfolio have developed a Market Impact Model and is incorporated into a software package known as the BARRA Aegis System. Market impact is calculated as a complicated function of the following terms.

- $V$  : the trade volume measured by value
- $\epsilon$  : the Elasticity. This measures the investors' responsiveness to asset price signals
- $\sigma$  : the volatility. This measures the uncertainty of the future price of an asset
- $\phi$  : the intensity. This forecasts near-term trading activity
- $\zeta$  : the distribution of trades sizes
- $\tau$  : the market tone. This measures the price of liquidity market-wide

The functional market impact cost function is quite complex. Accordingly they compute  $F(\epsilon, \sigma, \phi, \zeta, \tau)$  by replacing it with an approximation.

A typical institutional investment is usually so large that it will almost always be broken up into smaller trades and traded over a number of periods [12]. This style of trading faces the possibility of significant opportunity costs, due mainly to adverse price movements during the periods. Chan and Lakonishok [13] found that the costs associated with such trading styles face higher costs than individual trades. An individual

investor will have to assess whether the risk of the unperturbed price changing is greater or less than the price impact of trading a large number of shares in one transaction immediately.

The next section attempts to explain the main modelling ideas of theoretically minimising the expected cost of a block trade.

#### 4. A theoretical model to minimise the cost of block trading

This model simplifies the reality of the trading system and assumes one market maker who prices the shares as a function of the trade size and the current market conditions. The market maker does not price according to his or hers stock levels.

The problem is formulated in the following way. An investor decides to acquire a fixed number of shares ( $\bar{S}$ ) within  $T$  periods. (A period could be 30 minutes). A particular period is represented by  $t$ ,  $t = 1, 2, \dots, T$ .  $S_t$  represents the number of shares acquired in period  $t$  at price  $P_t$ . So this problem can be expressed as:

$$\min_{\{S_t\}} \left[ \sum_{t=1}^T P_t S_t \right] \quad (3)$$

$$\text{Subject to } \sum_{t=1}^T S_t = \bar{S} \quad (4)$$

$$S_t \geq 0, \quad t = 1, 2, \dots, T \quad (5)$$

Bertsimas [8], and Tamiz [34] offers a detailed explanation of this modelling procedure.

$W_t$  is defined to represent the number of shares outstanding at time  $t$ . Hence,  $W_1 = \bar{S}$  and  $W_{T+1} = 0$ .  $W_t$  can be obtained recursively for  $t = 2, \dots, T-1$ .  $E_1$  represents the expected cost of all the trades at period one.

$$W_t = W_{t-1} - S_{t-1}, \quad W_1 = \bar{S}, \quad W_{T+1} = 0.$$

Often in financial modelling, share prices are modelled using a Geometric Brownian motion, [20], [4]. This price dynamic is represented as  $\tilde{P}_t$ .

$$\tilde{P}_t = \tilde{P}_{t-1} \exp(Z_t)$$

where  $Z_t$  is an IID normal random variable with mean  $\mu_z$  and variance  $\sigma_z^2$ .

The execution price of each transaction  $P_t$  is a function of  $\tilde{P}_t$

(the noimpact price) and a price impact of the trade  $\Delta_t$ . Therefore:

$$P_t = \tilde{P}_t + \Delta_t$$

This price dynamic models the market timing cost and the price impact cost, thus it effectively models the execution cost. This model aims to minimise execution costs only and not transaction costs.

The no-impact price ( $\tilde{P}_t$ ) may be viewed as the price in the absence of any trade. The price impact trade function ( $\Delta_t$ ) has only a temporary effect on the execution price – because  $P_t$  does not affect  $P_{t+1}$ .  $\Delta_t$  must be a function of the number of shares traded and the market conditions at time  $t$ . The number of shares traded at time  $t$  is represented by  $S_t$ , and the market conditions by  $X_t$ . Therefore:

$$\Delta_t = (\alpha S_t + \gamma X_t) \tilde{P}_t$$

The presence of  $X_t$  in the execution price function  $P_t$ , enables the price impact function to reflect the potential impact of changing market conditions or private information about the security. For example,  $X_t$  might be the return on the FTSE 100 index. Generally, a rise in the FTSE 100 will affect the price of most securities to some degree, and  $\gamma$  measures the degree to which this particular security is affected by markets movements.  $\alpha$  measures the degree to which  $S_t$ , the size of the trade, affects the execution price  $P_t$ .

$X_t$  is obtained recursively by the following equation:

$$x_t = \rho X_{t-1} + \eta_t$$

where  $\eta_t$  is white noise with mean 0 and variance  $\sigma_\eta^2$ .  $X_t$  is an Arithmetic Random walk which captures varying degrees of predictability in information or market conditions.

#### 4.1 Computation required – dynamic programming

Bellmen's principle of optimality allows a recursive optimality equation to be developed, with an associated cost function. For this problem, the backward recursive optimality equation in its simplest form can be written as:

$$\text{cost}(t, W_t) = \min_{\{S_t\}} E_t[v(P_t, X_t, S_t) + \text{cost}(t + 1, W_t - S_t)] \quad (6)$$

where,  $v(P_t, X_t, S_t)$  represents the expected cost of accruing  $S_t$  shares under market conditions  $X_t$  and the unperturbed price  $\tilde{P}_t$ .  $\tilde{P}_t$ ,  $X_t$  are generated using a simulation technique and are discretised, meaning they can only take certain values.

Importantly

$$\text{cost}(T, W_T) = v(P_T, X_T, W_T)$$

$\text{cost}(T, W_T)$  is calculated for all values of  $W_T$  at time  $T$ . Namely  $W_T = 0, 1, 2, \dots, \bar{S}$ , and

$$\text{cost}(1, \bar{S}) = \min_{\{S_1\}} E_1[v(P_1, X_1, S_1) + \text{cost}(2, \bar{S} - S_1)] \quad (7)$$

$\text{cost}(1, \bar{S})$  is calculated for all possible values of  $S_1$ , namely  $S_1 = 0, 1, \dots, \bar{S}$ . The program selects the lowest cost and stores the optimal decision ( $S_1^*(\bar{S})$ ).

Back tracking through the system determines the optimal decisions for all of the possible states in periods  $t = 1, 2, \dots, T$ , namely ( $S_t^*(W_t)$ ). Forward tracking through the system determines the series of optimal decisions to achieve the minimum expected cost of purchasing  $\bar{S}$  within  $T$  periods.

To reduce the computation to a manageable size,  $S_t$  is discretised to fixed increments of  $s$  shares. A logical choice would be a lot size of 100.

#### 4.1.1 Parameter values

$$P_0 = \text{£} 50$$

$$\text{Minimum price} = \text{£} 47$$

$$\text{Maximum price} = \text{£} 53$$

$$\text{Price increments} = \text{£} 0.10$$

$$\mu_z = 0.0$$

$$\sigma_z = 0.02/\sqrt{13}$$

$$\alpha = 0.0000005$$

$$\eta_t \sim (0, 1 - \sigma^2)$$

$$\gamma = 0.01$$

$$\rho = 0.50$$

$$\bar{S} = 100,000$$

$$\text{Lot Size} = 100$$

$$\text{Number of lots to be purchased} = 1000$$

The price of the share prior to the transactions is  $\text{£} 50$ . The next two parameters imply that the continuously compounded  $\log \tilde{P}_t / \tilde{P}_{t-1}$  has zero mean and a 2% daily standard deviation. A period was assumed to be 30 minutes and there are 13 such periods in a typical trading day, hence the divisor of  $\sqrt{13}$ .

$\alpha$  is calibrated to yield a 5% price impact on a 100,000 share trade.

The variance of the error term  $\eta_t$  for the information variable  $X_t$  is a function of the autocorrelation coefficient  $\rho$  – this specification yields a unit variance for  $X_t$  for any value of  $\rho$ .

$\gamma$  measures the degree to which  $X_t$  affects the execution price,  $P_t$ .

When  $\rho = 0.0$ , this implies that  $X_{t-k}$  is not affected by the market condition in the previous period ( $X_{t-k-1}$ ). Therefore,  $X_{t-k}$  just equals the white noise  $\eta_{t-k}$ . In a sense  $X_{t-k}$  is unforecastable.

When  $\rho$  is greater than zero, the market impact in the current period is likely to exhibit the same trend as the previous period, a reversal in the trend is possible due the random white noise  $\eta_t$ . Conversely, if  $\rho$  is less than zero, the market impact in the current period is likely to exhibit an opposite trend to the trend in the previous period, again the trend could continue due to the white noise.

This paper considers only one combination of these parameters.

The simulations were performed using a Pentium 200 PC and simulations were written in Fortran. Some of the results are presented in the next section.

## 5. Results

As means of comparison, the expected cost of performing the optimal strategy is compared against the strategy of trading equally sized lots over the  $T$  periods, this style of trading is known as the Naive strategy. Obviously, each trade would consist of  $\bar{S}/T$  shares traded over all of the  $T$  periods.

This section provides two main sets of results. Firstly, the results from one simulation using  $\gamma = 0.01$  and  $\rho = 0.5$ . These results compare the optimal strategy and the naive strategy. Secondly, the results obtained from a Monte Carlo simulation using the same parameters (10,000 Replications).

Tamiz and Treloar [34] offers a more detailed discussion and a larger array of results for the discretised route.

The execution cost per share is calculated using the following equation:

$$\text{Execution Cost per share} = \frac{\text{Optimal Cost} - P_0 \bar{S}}{\bar{S}} \quad (8)$$

where  $P_0 = \text{£}50$  and  $\bar{S} = 100\,000$ .

This a simplified version of the pre-trade measure.

5.1 One simulation:  $\gamma = 0.01$  and  $\rho = 0.5$

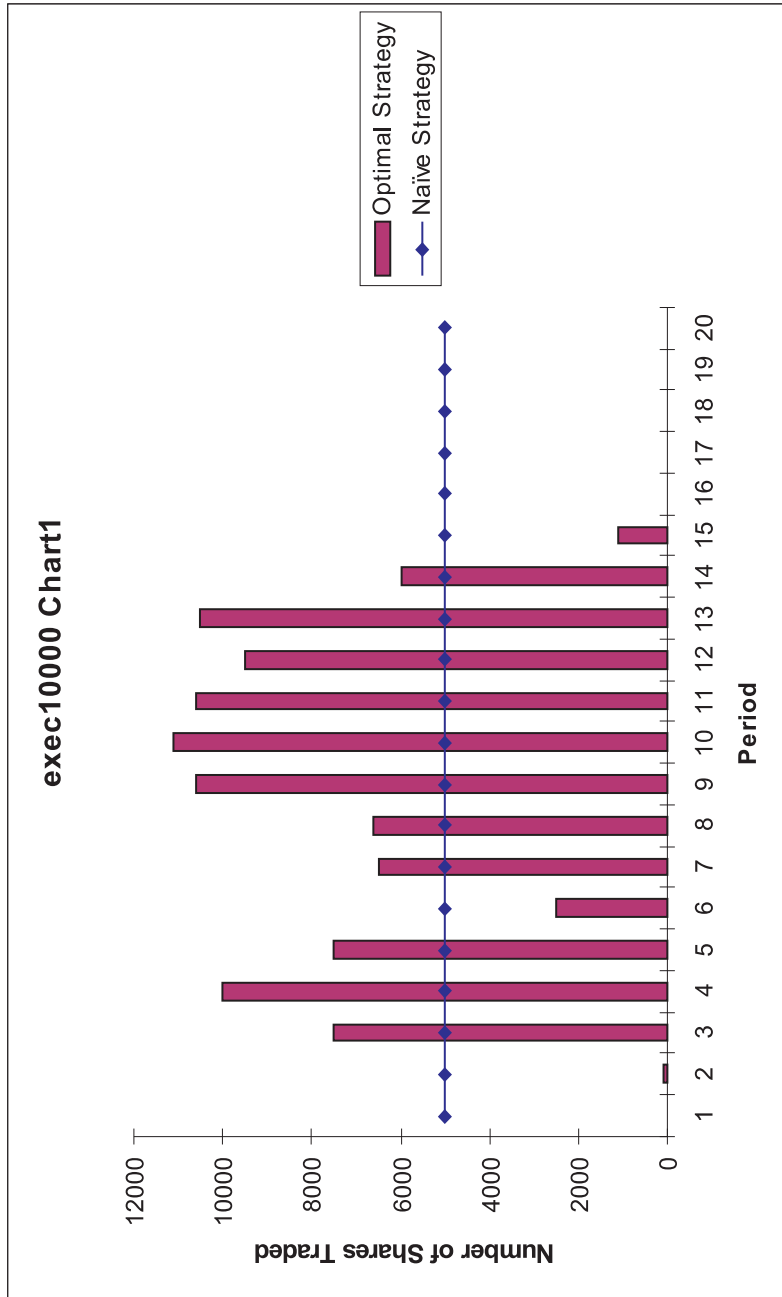


Figure 1

## 5.1.1 Naive strategy

| $t$ | $\tilde{P}_t$ | $X_t$ | $S_t$ | $\gamma X_t \tilde{P}_t$ | $\alpha S_t \tilde{P}_t$ | $P_t$  | cost( $t$ ) |
|-----|---------------|-------|-------|--------------------------|--------------------------|--------|-------------|
| 1   | £50.00        | 1.25  | 5000  | £0.625                   | £0.125                   | £50.75 | £253,750.00 |
| 2   | £50.10        | 0.75  | 5000  | £0.376                   | £0.125                   | £50.60 | £253,005.00 |
| 3   | £49.90        | -0.25 | 5000  | -£0.125                  | £0.125                   | £49.90 | £249,500.00 |
| 4   | £49.90        | -0.50 | 5000  | -£0.250                  | £0.125                   | £49.78 | £248,876.25 |
| 5   | £49.90        | -0.25 | 5000  | -£0.125                  | £0.125                   | £49.90 | £249,500.00 |
| 6   | £49.90        | 0.25  | 5000  | £0.125                   | £0.125                   | £50.15 | £250,747.50 |
| 7   | £49.70        | 0.25  | 5000  | £0.124                   | £0.124                   | £49.95 | £249,742.50 |
| 8   | £49.70        | 0.25  | 5000  | £0.124                   | £0.124                   | £49.95 | £249,742.50 |
| 9   | £49.50        | 0.25  | 5000  | £0.124                   | £0.124                   | £49.75 | £248,737.50 |
| 10  | £49.60        | 0.00  | 5000  | £0.000                   | £0.124                   | £49.72 | £248,620.00 |
| 11  | £49.50        | 0.25  | 5000  | £0.124                   | £0.124                   | £49.75 | £248,737.50 |
| 12  | £49.80        | -0.25 | 5000  | -£0.125                  | £0.125                   | £49.80 | £249,000.00 |
| 13  | £50.00        | -0.75 | 5000  | -£0.375                  | £0.125                   | £49.75 | £248,750.00 |
| 14  | £50.10        | -0.50 | 5000  | -£0.251                  | £0.125                   | £49.97 | £249,873.75 |
| 15  | £50.10        | 0.00  | 5000  | £0.000                   | £0.125                   | £50.23 | £251,126.25 |
| 16  | £50.20        | 0.50  | 5000  | £0.251                   | £0.126                   | £50.58 | £252,882.50 |
| 17  | £50.50        | 0.50  | 5000  | £0.253                   | £0.126                   | £50.88 | £254,393.75 |
| 18  | £50.50        | 0.50  | 5000  | £0.253                   | £0.126                   | £50.88 | £254,393.75 |
| 19  | £50.60        | 0.00  | 5000  | £0.000                   | £0.127                   | £50.73 | £253,632.50 |
| 20  | £50.90        | 0.25  | 5000  | £0.127                   | £0.127                   | £51.15 | £255,772.50 |

Total Cost of the Naive Strategy = £5,020,783.75

Execution Cost of the Naive Strategy = £20,783.75

Execution Cost per Share = £0.21

## 5.1.2 Optimal strategy

| $t$ | $\tilde{P}_t$ | $X_t$ | $S_t$ | $\gamma X_t \tilde{P}_t$ | $\alpha S_t \tilde{P}_t$ | $P_t$  | cost( $t$ ) |
|-----|---------------|-------|-------|--------------------------|--------------------------|--------|-------------|
| 1   | £50.00        | 1.25  | 0     | £0.0                     | £0.0                     | £0.0   | £0.0        |
| 2   | £50.10        | 0.75  | 100   | £0.376                   | £0.003                   | £50.48 | £5,047.83   |
| 3   | £49.90        | -0.25 | 7500  | -£0.125                  | £0.187                   | £49.96 | £374,717.81 |
| 4   | £49.90        | -0.50 | 10000 | -£0.250                  | £0.250                   | £49.90 | £499,000.00 |
| 5   | £49.90        | -0.25 | 7500  | -£0.125                  | £0.187                   | £49.96 | £374,717.81 |
| 6   | £49.90        | 0.25  | 2500  | £0.125                   | £0.062                   | £50.09 | £125,217.81 |
| 7   | £49.70        | 0.25  | 6500  | £0.124                   | £0.162                   | £49.99 | £324,907.54 |
| 8   | £49.70        | 0.25  | 6600  | £0.124                   | £0.164                   | £49.99 | £329,922.52 |
| 9   | £49.50        | 0.25  | 10600 | £0.124                   | £0.262                   | £49.89 | £528,792.66 |
| 10  | £49.60        | 0.00  | 11100 | £0.000                   | £0.275                   | £49.88 | £553,615.61 |
| 11  | £49.50        | 0.25  | 10600 | £0.124                   | £0.262                   | £49.89 | £528,792.66 |
| 12  | £49.80        | -0.25 | 9500  | -£0.125                  | £0.237                   | £49.91 | £474,164.48 |
| 13  | £50.00        | -0.75 | 10500 | -£0.375                  | £0.263                   | £49.89 | £523,818.75 |
| 14  | £50.10        | -0.50 | 6000  | -£0.251                  | £0.150                   | £50.00 | £299,998.80 |
| 15  | £50.10        | 0.00  | 1000  | £0.000                   | £0.025                   | £50.13 | £50,125.05  |
| 16  | £50.20        | 0.50  | 0     | £0.000                   | £0.000                   | £0.00  | £0.00       |
| 17  | £50.50        | 0.50  | 0     | £0.000                   | £0.000                   | £0.00  | £0.00       |
| 18  | £50.50        | 0.50  | 0     | £0.000                   | £0.000                   | £0.00  | £0.00       |
| 19  | £50.60        | 0.00  | 0     | £0.000                   | £0.000                   | £0.00  | £0.00       |
| 20  | £50.90        | 0.25  | 0     | £0.000                   | £0.000                   | £0.00  | £0.00       |

Total Cost of the Optimal Strategy = £4,992,839.32

Execution Cost of the Optimal Strategy = -£7,160.68

Execution Cost per Share = -£0.07

5.2 Ten thousand simulation's:  $\gamma = 0.01$  and  $\rho = 0.5$

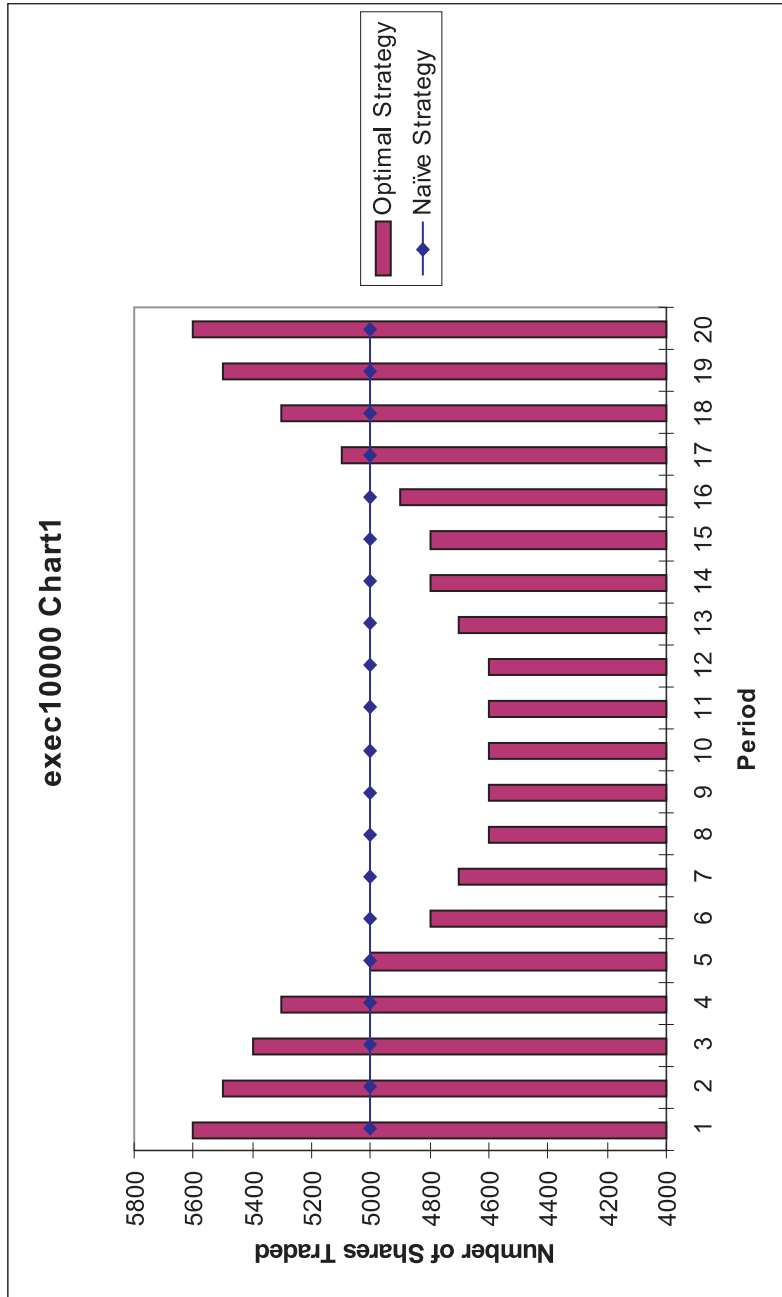


Figure 2

### 5.2.1 Naive strategy

Total Cost of the Naive Strategy = £ 5,024,233.18

Execution Cost of the Naive Strategy = £ 24,233.18

Execution Cost per Share = £ 0.24

### 5.2.2 Optimal strategy

Total Cost of the Optimal Strategy = £ 5,004,624.89

Execution Cost of the Optimal Strategy = £ 4,624.89

Execution Cost per Share = £ 0.046

## 6. Conclusion

The work by Bertsimas [8] demonstrates the difficulty in modelling and minimising execution costs, let alone transaction costs. The results have clearly shown that under these market modelling assumptions the Naive strategy is not an optimal course of action. Bertsimas [8] proved that the Naive strategy is only optimal when the price impact function is linear without a market information component.

$$P_t = P_{t-1}(1 + \alpha S_t) \quad (9)$$

Once you build in the reality of an information component into the price impact function, the optimal strategy is unlikely to be the naive strategy. Our results have shown the optimal strategy consistently produced lower execution costs than the naive strategy. Also, the optimal trading strategy often quite different to the naive trading strategy.

This paper has discussed the difficulty of defining, measuring and minimising transaction costs. Estimating transaction costs should alter investment managers portfolio decisions. Portfolio managers will often not achieve desired optimal portfolio's, because true transaction costs are not considered. For example, once the managers decide on the target portfolio, the trading desk begins implementing the trades. As the trading progresses, market impact is observed to drive up the price of some assets. The manager then decides either to pay or to abandon the attempted trades. Generally these decisions will be made on an asset-by-asset basis, so a portfolio viewpoint is lost. The portfolio that finally emerges from the trading process can differ from the original optimal portfolio in a fairly ad hoc way. The weakness in the process occurs due to transaction costs were not incorporated in the portfolio construction at

the start. The problem then arises, how do we measure and importantly predict these costs?

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