Applying interactive fuzzy multi-objective linear programming to transportation planning decisions

Tien-Fu Liang*

Department of Industrial Engineering and Management
Hsiuping Institute of Technology
11 Gungye Road, Dali City
Taichung
Taiwan 412
R.O.C.

Abstract

In real-world transportation planning decision (TPD) problems, input data or related parameters are frequently imprecise/fuzzy owing to incomplete or unobtainable information. This work presents a novel interactive fuzzy multi-objective linear programming (*i*-FMOLP) model for solving TPD problems with multiple fuzzy objectives. The proposed *i*-FMOLP model attempts to minimize simultaneously the total production and transportation costs and the total delivery time with reference to available capacities at each source and forecast demand at each destination. An industrial case study shows that the proposed *i*-FMOLP model yields an efficient compromise solution and the overall DM levels of satisfaction with determined objective values. In addition, the proposed model provides a systematic framework that facilitates decision-making, enabling a DM to interactively modify the fuzzy data and parameters until obtained a satisfactory solution. Overall, the proposed *i*-FMOLP model is practically applicable for solving TPD problems.

Keywords : *Transportation planning decisions, interactive fuzzy multi-objective linear programming, fuzzy set theory.*

1. Introduction

The transportation planning decision (TPD) problem involves the distribution of goods and services from a set of sources (e.g. factories) to a

**E-mail*: farmer@mail.hit.edu.tw

Journal of Information & Optimization Sciences
Vol. 27 (2006), No. 1, pp. 107–126
 0252-2667/06 $2.00 + 0.25

set of destinations (e.g. warehouses). With a variety of transporting routes and differing transportation costs for the routes, the aim is to determine how many units should be shipped from each source to each destination so that all demands are satisfied with the minimum total transportation costs. Basically, the TPD problem is a special type of a linear programming (LP) problem that can be solved using the standard simplex method. Some special solution algorithms, such as the stepping stone method and the modified distribution method, allow TPD problems to be solved much more easily than the general LP method. However, when any of the LP method or the existing effective algorithms is used to solve the TPD problems, the goal and related inputs are generally assumed to be deterministic/crisp [25].

In most practical TPD problems, input data or related parameters are often imprecise/fuzzy owing to incomplete or unobtainable information. Obviously, conventional LP method and solution algorithms cannot solve all fuzzy TPD problems. In 1976, Zimmermann [27] first introduced fuzzy set theory into an ordinary LP problem with fuzzy goal and constraints. Following the fuzzy decision-making method proposed by Bellman and Zadeh [2], that study confirmed the existence of an equivalent LP problem. Since then, fuzzy linear programming (FLP) has developed into several fuzzy optimization methods for solving TPD problems. Chanas et al. [6] presented an FLP model for solving the TPD problem with crisp cost coefficients and fuzzy supply and demand values. Moreover, Chanas and Kuchta [8] proposed the concept of the optimal solution of the TPD problem with fuzzy coefficients expressed as L-L fuzzy numbers, and developed an algorithm for obtaining the solution. Related works on the use of FLP to solve TPD problems included Bit et al. [3], Chanas et al. [7], and Chanas and Kuchta [9].

However, in real-world TPD problems, the decision maker (DM) must simultaneously handle conflicting aims that govern the use of the resources within organizations. These aims are minimizing total production costs, total transportation costs and total delivery time/distance, and maximizing total profits, total relative safety, customer service level and utilization of equipment and facilities [1, 11-13, 17, 20, 23]. Particularly, the DM must simultaneously optimize these conflicting objectives in a framework of fuzzy aspiration levels. In 1978, Zimmermann [28] first extended his FLP approach to a conventional multi-objective linear programming (MOLP) problem. For each of the objective functions in this problem, the DM was assumed to have a fuzzy objective, such as "the objective function should be substantially less than or equal to some value". Subsequent works on fuzzy goals programming (FGP) included Leberling [19], Hannan [15], Luhandjula [22], Sakawa [24], Kuwano [18], Chen and Tsai [10], and Wang and Liang [26].

Subsequently, researches have developed several FOP methods to solve multi-objective TPD problems. Bit et al. [5] proposed an additive fuzzy programming model that considered weights and priorities for all non-equivalent objectives for the multi-objective TPD problem. Li and Lai [21] designed a fuzzy compromise programming method to obtain a non-dominated compromise solution for multi-objective TPD problems in which various objectives were synthetically considered with the marginal evaluation for individual objectives and the global evaluation for all objectives. El-Wahed [14] developed a fuzzy programming approach to determine the optimal compromise solution of a multi-objective TPD problem by measuring the degree of closeness of the compromise solution to the ideal solution using a family of distance functions. Related works on fuzzy multi-objective TPD programming problems included Bit et al. [4] and Hussein [16]. This work develops a novel interactive fuzzy multi-objective linear programming (*i*-FMOLP) model for solving TPD problems with multiple fuzzy objectives. The proposed *i*-FMOLP model attempts to simultaneously minimize the total production and transportation costs and the total delivery time with reference to available capacities at each source and forecast demand at each destination. The rest of this work is organized as follows. Section 2 describes the problem, details the assumptions and formulates the problem. Section 3 then develops the *i*-FMOLP model and details the solution procedure for solving TPD problems with fuzzy multiple objectives. Subsequently, Section 4 presents an industrial case designed to implement the feasibility of applying the proposed model to real TPD problems. Next, Section 5 discusses the findings for the practical application of the proposed *i*-FMOLP model. Conclusions finally are drawn in Section 6.

2. Problem formulation

2.1 *Problem description, assumptions and notation*

The TPD problem examined herein can be described as follows. Assume that a distribution center seeks to determine the transportation plan of a homogeneous commodity from m sources to n destinations. Each source has an available supply of the commodity to distribute to various destinations, and each destination has a forecast demand of the commodity to be received from the sources. The TPD proposed herein attempts to determine the optimal volumes to be transported from each source to each destination to simultaneously minimize the total production and transportation costs and the total delivery time. The TPD problem proposed in this work focuses on developing an interactive *i*-FMOLP model for optimizing the transportation plan in fuzzy environments.

The mathematical model developed herein is based on the following assumptions.

(1) All of the objective functions are fuzzy with imprecise aspiration levels.

(2) All of the objective functions and constraints are linear equations.

(3) The values of all model parameters are certain over the planning horizon.

(4) The transportation costs and delivery time on a given route is directly proportional to the units shipped.

(5) The total supply available at all sources just equals the total demand required at all destinations.

(6) The linear membership functions are assigned to represent the fuzzy sets involved, and the minimum operator is used to aggregate all fuzzy sets.

Assumption 1 concerns the fuzziness of the objective functions in practical TPD problems, and incorporates the variations in the DM's judgments for the solutions of fuzzy multi-objective optimization problems in a framework of fuzzy aspiration levels. Assumptions 2 to 4 indicate that the linearity, certainty and proportionality properties must be technically satisfied as a standard LP problem. Assumption 5 is the 'necessary and sufficient' condition for a feasible solution to the TPD problem. Assumption 6 is made to convert the original fuzzy MOLP problem into an equivalent LP problem that can be solved efficiently by the standard simplex method [30].

2.2 *Problem formulation*

2.2.1 Objective functions

This work chose the multi-objective functions for solving the TPD problem by reviewing the literature and considering practical situations. Diaz [13] specified three objective functions to minimize the total transportation costs, total delivery time, and total relative time for a TPD problem with fuzzy multiple objectives. Practical TPD problems typically minimized the total production costs, total transportation costs and total delivery time [5, 12, 21, 23]. Accordingly, two objective functions were simultaneously considered in developing the proposed MOLP model, as follows.

- *Minimize total production and transportation costs*

$$\text{Min}\, z_1 \cong \sum_{i=1}^{m} \sum_{j=1}^{n} (p_{ij} + c_{ij}) Q_{ij} \tag{1}$$

- *Minimize total delivery time*

$$\text{Min}\, z_2 \cong \sum_{i=1}^{n} \sum_{j=1}^{m} t_{ij} Q_{ij} \tag{2}$$

where

z_1	total production and transportation costs ($)
z_2	total delivery time (hours)
Q_{ij}	units transported from source i to destination j (units)
p_{ij}	production cost per unit from source i to destination j ($/unit)
c_{ij}	transportation cost per unit from source i to destination j ($/unit)
t_{ij}	transportation time per unit from source i to destination j ($/unit)

The symbol '$\cong$' is the fuzzified version of '$=$' and refers to the fuzzification of the aspiration levels. In real-world TPD problems, the environmental coefficients and operation parameters are usually uncertain because some information is incomplete or unobtainable over the planning horizon. Accordingly, Equations. (1) and (2) are fuzzy with imprecise aspiration levels, and incorporate the variations in the DM's judgments regarding the solutions of fuzzy multi-objective optimization problems. For each of the objective functions of the proposed MOLP model, this work assumes that the DM has such imprecise objectives as, "the objective functions should be essentially equal to some value". These conflicting objectives are required to be simultaneously optimized by the DM in the framework of fuzzy aspiration levels.

2.2.2 Constraints

- *Constraints on total supply available for each source i*

$$\sum_{j=1}^{n} Q_{ij} = S_i\,, \quad i = 1, 2, \ldots, m \tag{3}$$

- *Constraints on total demand for each destination j*

$$\sum_{i=1}^{n} Q_{ij} = D_i\,, \quad i = 1, 2, \ldots, n \tag{4}$$

- *Non-negativity constraints on decision variables*

$$Q_{ij} \geq 0\,, \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \tag{5}$$

where S_i denotes the total supply available of source i (units), and D_j denotes the total demand of destination j (units). This work addresses a practical application of a FGP model for solving the TPD problem with fuzzy multiple objectives. Therefore, the constraints (3) and (4) in the proposed MOLP model are assumed to be crisp. Notably, the MOLP model described above has a feasible solution only if the total supply available at all sources just equals the total demand required at all destinations.

3. Model development

The original MOLP model above can be converted to the *i*-FMOLP model using the Zimmermann's fuzzy programming method [28] and the fuzzy decision-making of Bellman and Zadeh [2]. The linear membership functions and the minimum operator are adopted to aggregator the original fuzzy MOLP problem into an ordinary LP problem.

3.1 *Linear membership functions*

The corresponding linear membership function for each fuzzy objective function is defined by

$$f_g(z_g) = \begin{cases} 1 & z_g \leq z_g^l \\ \dfrac{z_g^u - z_g}{z_g^u - z_g^l} & z_g^l < z_g < z_g^u\,, \quad g = 1, 2, \ldots, k \\ 0 & z_g \geq z_g^u \end{cases} \tag{6}$$

where z_g^l and z_g^u are the lower and upper bounds, respectively, of the gth objective function z_g. The linear membership function can be determined by requiring the DM to select the objective values interval $[z_g^l, z_g^u]$. In practical situations, a possible interval for imprecise objective values can be estimated based on the experience and knowledge of DM or experts, and their equivalent membership values of the DM in the interval $[0, 1]$. Figure 1 shows the graph of the linear membership function for Equation (6).

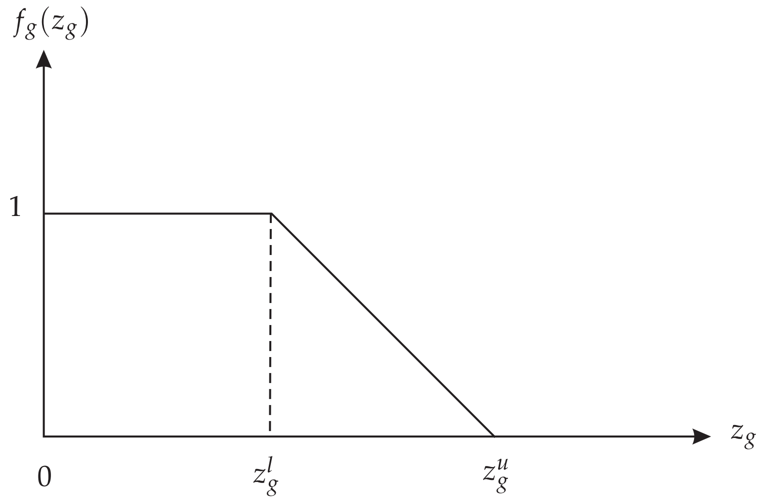

Figure 1
Linear membership function

3.2 *Fuzzy decision of Bellman and Zadeh* [2]

Let X be a given set of all possible solutions to a decision problem. A fuzzy goal G is a fuzzy set on X characterized by its membership function

$$\mu_G : X \to [0, 1]\,. \tag{7}$$

A fuzzy constraint C is a fuzzy set on X characterized by its membership function

$$\mu_C : X \to [0,1] . \tag{8}$$

Then, G and C combine to generate a fuzzy decision D on X, which is a fuzzy set resulting from intersection of G and C, and is characterized by its membership function

$$L = \mu_D(x) = \mu_G(x) \wedge \mu_C(x) = \mathrm{Min}(\mu_G(x), \mu_C(x)) \tag{9}$$

and the corresponding maximizing decision is defined by

$$\mathrm{Max}\, L = \mathrm{Max}\, \mu_D(x) = \mathrm{Max}\,\mathrm{Min}(\mu_G(x), \mu_C(x)) . \tag{10}$$

More generally, suppose the fuzzy decision D results from k fuzzy goals $G_1, \ldots, G_k$ and m constraints $C_1, \ldots, C_m$. Then the fuzzy decision D is the intersection of $G_1, \ldots, G_k$ and $C_1, \ldots, C_m$, and is characterized by its membership function

$$\begin{aligned} L &= \mu_D(x) \\ &= \mu_{G_1}(x) \wedge \mu_{G_2}(x) \wedge \ldots \wedge \mu_{G_k} \wedge \mu_{C_1} \wedge \mu_{C_2} \wedge \ldots \wedge \mu_{C_m} \\ &= \mathrm{Min}(\mu_{G_1}(x), \mu_{G_2}(x), \ldots, \mu_{G_k}(x), \mu_{C_1}(x), \mu_{C_2}(x), \ldots, \mu_{C_k}(x)) \end{aligned} \tag{11}$$

and the corresponding maximizing decision is defined by

$$\begin{aligned} \mathrm{Max}\, L &= \mathrm{Max}\, \mu_D(x) \\ &= \mathrm{Max}\,\mathrm{Min}(\mu_{G_1}(x), \mu_{G_2}(x), \ldots, \mu_{G_k}(x), \mu_{C_1}(x), \ldots, \mu_{C_m}(x)) . \end{aligned} \tag{12}$$

3.3 *Interactive fuzzy multi-objective linear programming* (*i-FMOLP*) *model*

After the linear membership functions for all objective functions are specified, the minimum operator of the fuzzy decision-making of Bellman and Zadeh [2] is used to aggregate all fuzzy sets. Consequently, the original fuzzy MOLP model in the previous section can be converted into an equivalent single-objective LP problem that can be solved efficiently using the standard simplex method. The complete *i*-FMOLP model proposed herein for solving TPD problems can be formulated as follows.

$$\begin{aligned} &\mathrm{Max}\, L \\ &\text{subject to } \; L \le f_g(z_g) , \quad g = 1, 2, \ldots, k \\ &\qquad\qquad \sum_{j=1}^{n} Q_{ij} = S , \quad i = 1, 2, \ldots, m \end{aligned}$$

$$\sum_{i=1}^{m} Q_{ij} = D_j\,, \quad j = 1,2,\ldots,n$$

$$Q_{ij} \geq 0\,, \quad i = 1,2,\ldots,m,\ j = 1,2,\ldots,n$$

where the auxiliary variable L represents the overall degree of DM satisfaction with the determined multiple objective values.

The interactive solution procedure of the proposed *i*-FMOLP model for solving the multi-objective TPD problem is as follows.

Step 1. Formulate the original fuzzy MOLP model for the TPD problems.

Step 2. Specify the corresponding linear membership functions for all of the objective functions.

Step 3. Introduce the auxiliary variable L, and aggregate the original fuzzy MOLP problem into an equivalent ordinary single-objective LP (Max L) model using the minimum operator.

Step 4. Solve the LP problem and obtain the initial compromise solution.

Step 5. Execute and modify the interactive decision process. If the DM is not satisfied with the initial compromise solution, the model must be changed until a satisfactory solution is found.

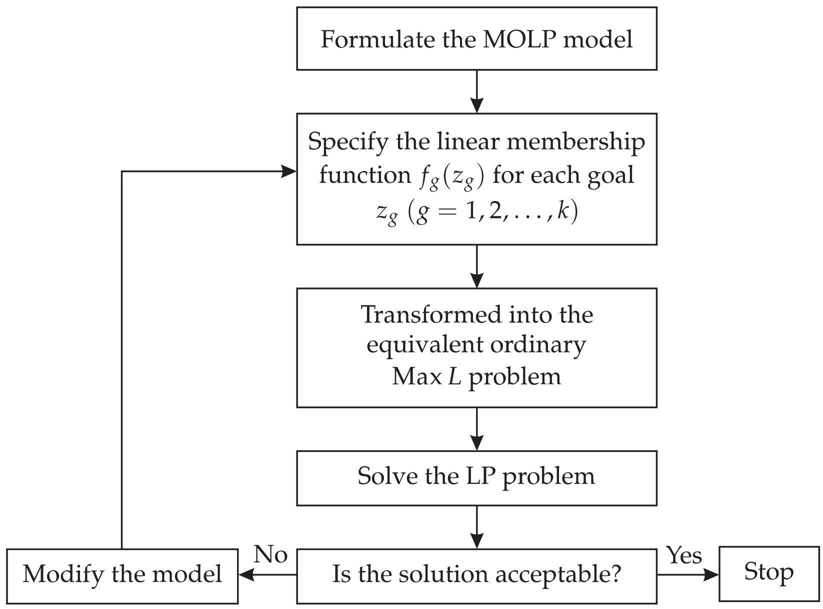

Figure 2
The block diagram of the proposed *i*-FMOLP model development

Figure 2 presents the block diagram of the proposed *i*-FMOLP model development for solving fuzzy multi-objective TPD problems.

4. Model implementation

4.1 *Data description*

Dali Company was used as a case study to demonstrate the practicality of the proposed methodology. Dali Company is the leading producer of soft drinks and low-temperature foods in Taiwan. Currently, Dali plans to develop the South-East Asian market and broaden the visibility of Dali products in the Chinese market. Notably, following the entry of Taiwan to the World Trade Organization, Dali plans to seek strategic alliances with well-known international companies, and previously introduced international bread to lighten the embedded future influence of this strategy. In the domestic soft drinks market, Dali produces tea beverages to satisfy demand from five distribution centers in Taichung, Chiayi, Kaohsiung, Taipei and Haulien, with production based at three factories in Changhua, Touliu and Hsinchu. Table 1 summarizes the basic data of the Dali case for the upcoming season. Listed on the right-hand side of the matrix is the potential supply available at each factory, and on the bottom is the forecast demand at each distribution center. Each cell in the matrix represents a route from particular factory to a particular distribution center. Each cell shows the transportation costs and delivery time of shipping one unit (in thousand dozen bottles) through a particular route. For example, as illustrated in Table, the available supply of the Changhua factory is 18000 units, the forecast demand of the Taichung distribution center is 10000 units, and the transportation costs and delivery time per unit from Changhua to Tacchung are \$10 and 6 hours, respectively. The production cost per dozen bottles is Changhua, \$15; Touliu, \$10, and Hsinchu, \$8.

Since transportation costs are a major expense, Dali's management is initiating a study to reduce these costs as much as possible. The TPD problem for the industrial case presented herein concentrates on developing an *i*-FMOLP model for optimizing the transportation plan in fuzzy environments. The expected objectives of solving the TPD problem are to simultaneously minimize the total production and transportation costs and the total delivery time subject to the constraints on the row and column totals.

Table 1
Summarized data in the Dali case (in U.S. dollar)

Source	Destination					Supply (in 000 dozen bottles)
	Taichung	Chiayi	Kaohsiung	Taipei	Haulien	
Changhua	\$10/6 hours	\$12/8 hours	\$16/12 hours	\$20/16 hours	\$30/40 hours	18
Touliu	\$12/10 hours	\$7/10 hours	\$13/15 hours	\$24/22 hours	\$36/32 hours	24
Hsinchu	\$14/12 hours	\$16/16 hours	\$20/18 hours	\$10/10 hours	\$32/30 hours	10
Demand (in 000 dozen bottles)	10	8	12	16	6	52

4.2 *Solution procedure*

The interactive solution procedure using the proposed *i*-FMOLP model for the Dali case is described as follows.

Step 1. Formulate the original fuzzy MOLP model for the TPD problem according to Equations (1) to (5).

Step 2. Specify the corresponding linear membership functions for all of the objective functions. First, find the initial solutions for each of the objective functions by solving the crisp single-objective LP model. The results are $z_1 = \$1,310,000$ and $z_2 = 702$ hours. Moreover, the corresponding linear membership functions for each objective function can be defined using Equation (6), as below.

$$f_1(z_1) = \begin{cases} 1 & z_1 \leq 1200000 \\ \dfrac{2400000 - z_1}{1200000} & 1200000 < z_1 < 2400000 \\ 0 & z_1 \geq 2400000 \end{cases} \tag{13}$$

$$f_2(z_2) = \begin{cases} 1 & z_2 \leq 600 \\ \dfrac{2000 - z_2}{1400} & 600 < z_2 < 2000 \\ 0 & z_2 \geq 2000 \end{cases} \tag{14}$$

Step 3. Introduce the auxiliary variable L, and convert the original fuzzy MOLP problem into an equivalent ordinary single-objective LP model using the minimum operator. Accordingly, the complete *i*-FMOLP model for solving the TPD problem for the Dali case can be formulated based on the crisp LP model as proposed in the previous section.

Step 4. Solve the ordinary LP problem and obtain the compromise solution. LINDO computer software is used to run this the *i*-FMOLP model. Table 2 lists the optimal TPD plans for the Dali case based on the current data. Using *i*-FMOLP to simultaneously minimize the total production and transportation costs (z_1) and the total delivery time (z_2), the results are $z_1 = \$1,310,000$, $z_2 = 702$ hours, and the overall level of DM satisfaction with determined objective values is 0.9271.

Table 2
Optimal transportation plan in the Dali case with *i*-FMOLP model

Item	Solutions
Q_{ij} (in 000 dozen bottles)	$Q_{11} = 10$, $Q_{12} = 0$, $Q_{13} = 2$, $Q_{14} = 6$, $Q_{15} = 0$, $Q_{21} = 0$, $Q_{22} = 8$, $Q_{23} = 10$, $Q_{24} = 0$, $Q_{25} = 6$, $Q_{31} = 0$, $Q_{32} = 0$, $Q_{33} = 0$, $Q_{34} = 10$, $Q_{34} = 0$,
Objective values	$L = 0.9271$ $z_1 = \$1,344,000$ $z_2 = 702$ hours

Step 5. Execute and modify the interactive decision process. The DM may attempt to interactively modify the results by adjusting the linear membership functions and related model parameters until a satisfactory solution is obtained.

5. Computational analysis

5.1 *Analysis of the results*

Several significant management implications for the practical application of the *i*-FMOLP model proposed here are as follows.

1. The proposed *i*-FMOLP model yields a set of efficient compromise solutions. Alternatively, the TPD problem of the Dali case presented above was solved with the crisp ordinary LP model. Table 3 compares the solutions from the single objective LP model with that from the proposed *i*-FMOLP model. As illustrated in Table 3, using LP-1 to minimize the total production and transportation costs led to optimal value and total delivery time of \$1,310,000 and 772 hours, respectively. Applying LP-2 to minimize the total delivery time, the optimal value

and the total production and transportation costs were $1,344,000 and 702 hours, respectively. In contrast with the proposed *i*-FMOLP model, the optimal results were $z_1 = \$1,344,000$ and $z_2 = 702$ hours. These figures show that the *i*-FMOLP results are a set of efficient compromise solutions with multiple fuzzy objectives, compared to the optimal values calculated by LP-1 and LP-2, respectively.

Table 3
Comparison of the crisp LP and the proposed *i*-FMOLP solutions

Item	LP-1	LP-2	The proposed *i*-FMOLP
Objective function	Min z_1	Min z_2	Max L
L (DM's overall degree of satisfaction)	100%	100%	92.71%
z_1 (Total production and transportation cost)	$1,310,000	$1,344,000	$1,344,000
z_2 (Total delivery time)	772 hours	702 hours	702 hours

2. The proposed *i*-FMOLP model provides the overall degree ofDM satisfaction with the determined multiple objective values. If the solution is $L = 1$, then each objective is fully satisfied; if $0 < L < 1$, then all of the objectives are satisfied at the level of L; and if $L = 0$, then none of the objectives are satisfied. For example, the overall degree of DM satisfaction with the objective values in the Dali case, $z_1 = \$1,344,000$ and $z_2 = 702$ hours, initially was generated as 0.9271. Moreover, the DM may attempt to adjust L value by modifying the membership degree of the fuzzy objective functions and related parameters to seek a set of better compromise solutions.
3. The DM generally faces a planning problem with multiple fuzzy objectives, when making a TPD decision. For most real-world TPD problems, the aims are to minimize the total production costs, total transportation costs and total delivery time/distance, and maximizing total profits, total relative safety, customer service level and utilization of equipment and facilities. Additionally, the environmental coefficients and related parameters of the TPD problem are typically uncertain because some information is incomplete or unobtainable over the planning horizon. The comparisons in Table 3 reveal the interaction of trade-offs and conflicts among dependent objective functions. Accordingly,

the proposed *i*-FMOLP model can satisfy the requirement for the practical application since it attempts to simultaneously minimize the total production and transportation costs and the total delivery time.

4. The specific membership value for each of the objective functions strongly affects the overall level of DM satisfaction. Alternatively, setting $f_1(z_1)$ to their original values in the Dali case implements the sensitivity by varying only the upper bounds of the membership function $f_2(z_2)$. Table 4 presents the implementation results for the L and objective values. The results indicate that the specific membership value for each of the objective functions strongly affects the overall level of DM satisfaction. This finding implies that the DM must specify an appropriate membership value range for each fuzzy objective function to yield an efficient satisfactory solution for the TPD problem. In practice, the crisp single-objective LP solutions frequently served as a starting point of the lower and upper bounds of the objective values, and both intervals must cover the LP solutions. For example, two objective functions of the original fuzzy MOLP problem presented in the Dali case were solved using the crisp single-objective LP model, and the corresponding objective values intervals of the linear membership functions are listed in Table 5.

Table 4

Results of implementing the upper bounds of the membership function $f_2(z_2)$

Item	Run 1	Run 2	Run 3	Run 4	Run 5
z_2^u	800 hours	1400 hours	2000 hours	2600 hours	3200 hours
L	0.4900	0.8725	0.9271	0.9490	0.9608
z_1	\$1,344,000	\$1,344,000	\$1,344,000	\$1,344,000	\$1,344,000
z_2	702 hours	702 hours	702 hours	702 hours	702 hours

Table 5

The values intervals of fuzzy objective functions

Item	LP-1	LP-2	(z_g^l, z_g^u) $(g = 1, 2)$
Objective function	Min z_1	Min z_2	Max L
z_1	\$1,310,000	—	(\$1,200,000, \$2,400,000)
z_2	—	702 hours	(600 hours, 2000 hours)

5. The *i*-FMOLP model developed here is constructed using the linear membership function to represent the fuzzy goals of the DM in the original fuzzy MOLP model, together with the minimum operator to aggregate all fuzzy sets. Moreover, the original fuzzy MOLP problem can be converted into an equivalent ordinary LP problem and is easily solved by the standard simplex method. Aggregation operators can be roughly classified into three categories: intersection, union, and averaging operators. Table 6 compares the common aggregation operators [29]. The minimum operator used in this work is preferable when the DM intends to make the optimal membership function values approximately equal or when the DM considers that the minimum operator is an approximate representation. However, for some practical situations, the use of the aggregation operator to draws maps above the maximum operator and below the minimum operator is important. Alternatively, the averaging operators consider the relative importance of fuzzy sets and have the compensative property so that the result of combination is medium [21, 26, 29].

5.2 *Model comparisons*

The proposed *i*-FMOLP model is practically applicable for solving TPD problems and can produce better decisions than other models. Table 7 compares the proposed *i*-FMOLP to the Chanas and Kuchta [8] and Li and Lai [21] models. Various features distinguish the proposed *i*-FMOLP model from other TPD models. First, the proposed model provides a systematic framework that facilitates decision-making process, enabling the DM to interactively modify the fuzzy data and related parameters until a set of satisfactory solutions is obtained. Second, the proposed model meets the requirements for practical application because it simultaneously minimizes the total production and transportation costs and the total delivery time, and can determine the DM's overall degree of satisfaction given these solved objective values. Third, the proposed model outputs more wide-ranging decision information than other models. The proposed model focuses on the TPD problem with fuzzy multiple objectives, and can also provide information on alternative strategies with reference to the imprecise objective functions, available capacities at each source, and forecast demand at each destination in uncertain environments. Finally, the proposed *i*-FMOLP model provides greater computational efficiency

Table 6
Comparisons of common aggregation operators

Operator	Example	Brief description
Intersection (t-norms)	• Minimum • Algebraic product • Bounded sum • Drastic intersection	• An aggregation scheme is implemented where fuzzy sets are connected by a logical 'and' • The result of combination is high if and only if all values are high • The minimum operator is a greatest t-norm
Union (t-conorms)	• Maximum • Algebraic sum • Bounded difference • Drastic union	• An aggregation scheme is implemented where fuzzy sets are connected by a logical 'or' • The result of combination is high if some values are high • The minimum operator is a smallest t-conorm
Averaging (Compensative)	• Mean • Weighted • γ • OWA (The ordered weighted averaging)	• Have the compensative property so that the result of combination will be medium • Consider the relative importance of the fuzzy sets • The γ-operator is the convex combination of the min-operator and the max-operator • OWA enables a DM to specify linguistically his agenda for aggregating a collection of fuzzy sets

by employing the linear membership functions to represent the fuzzy objectives of the DM, and results in more flexible doctrines through an interactive decision-making process.

6. Conclusions

In real-world transportation decision problems, the DM must simultaneously handle multiple conflicting objectives that govern the use of the resources within organizations, and input data or related parameters are

Table 7
Comparisons of the major TPD models

Factor	Chanas and Kuchta [8]	Li and Lai [21]	The proposed *i*-FMOLP
Objective function	Single	Multiple	Multiple
Objective value	Fuzzy	Fuzzy	Fuzzy
Constraints property	Crisp	Crisp	Crisp
DM's overall level of satisfaction	Not considered	Considered	Considered
Available capacities	Crisp	Crisp	Crisp
Forecast demand	Crisp	Crisp	Crisp
Unit costs coefficients	Fuzzy	Crisp	Crisp
Aggregate operator	—	Weighted root-power mean	Minimum
Revised flexibility	Medium	Medium	High
Computational efficiency	Medium	Low	High

frequently imprecise/fuzzy owing to incomplete or unobtainable information. Conventional mathematical programming techniques and algorithms clearly cannot solve related fuzzy multi-objective TPD problems. This work presents an interactive fuzzy multi-objective linear programming (*i*-FMOLP) model for solving TPD problems with multiple fuzzy goals. The proposed *i*-FMOLP model attempts to simultaneously minimize the total production and transportation costs and the total delivery time with reference to available capacities at each source and forecast demand at each destination. An industrial case demonstrates the feasibility of applying the proposed approach to a real transportation decision problem. Consequently, the proposed *i*-FMOLP model yields an efficient compromise solution and the overall DM levels of satisfaction with determined objective values.

In addition, the proposed model provides a systematic framework that facilitates decision-making, enabling the DM to interactively modify the fuzzy data and related model parameters until a satisfactory solution is found. Especially, several significant management implications and features of the proposed model are presented that distinguish it from the existing major TPD models. On the whole, the proposed *i*-FMOLP model may provide greater computational efficiency and more flexible doctrines than the conventional crisp LP and fuzzy programming approaches in a fuzzy environment. Accordingly, the proposed *i*-FMOLP model is practically applicable for solving TPD problems with multiple fuzzy objectives.

References

[1] Y. P. Aneja and K. P. K. Nair (1979), Bicriteria transportation problems, *Management Science*, Vol. 25, pp. 73–78.

[2] R. E. Bellman and L. A. Zadeh (1970), Decision-making in a fuzzy environment, *Management Science*, Vol. 17, pp. 141–164.

[3] A. K. Bit, M. P. Biswal and S. S. Alam (1992), Fuzzy programming approach to multicriteria decision making transportation problem, *Fuzzy Sets and Systems*, Vol. 50, pp. 35–41.

[4] A. K. Bit, M. P. Biswal and S. S. Alam (1993), Fuzzy programming approach to multiobjective solid transportation problem, *Fuzzy Sets and Systems*, Vol. 57, pp. 183–194.

[5] A. K. Bit, M. P. Biswal and S. S. Alam (1993), An additive fuzzy programming model for multiobjective transportation problem, *Fuzzy Sets and Systems*, Vol. 57, pp. 313–319.

[6] S. Chanas, W. Kolodziejczyk and A. Machaj (1984), A fuzzy approach to the transportation problem, *Fuzzy Sets and Systems*, Vol. 13, pp. 211–222.

[7] S. Chanas, M. Deigado, J. L. Verdegay and M. A. Vila (1993), Interval and fuzzy extension of classical transportation problem, *Transportation Planning Technology*, Vol. 17, pp. 203–218.

[8] S. Chanas and D. Kuchta (1996), A concept of the optimal solution of the transportation problem with fuzzy cost coefficients, *Fuzzy Sets and Systems*, Vol. 82, pp. 299–305.

[9] S. Chanas and D. Kuchta (1998), Fuzzy integer transportation problem, *Fuzzy Sets and Systems*, Vol. 98, pp. 291–298.

[10] L. H.Chen and F. C. Tsai (2001), Fuzzy goal programming with different important and priorities, *European Journal of Operational Research*, Vol. 133, pp. 548–556.

[11] J. N. Clirnaco, C. H. Antunes and M. J. Alves (1993), Interactive decision support for multi-objective transportation problem, *European Journal of Operational Research*, Vol. 65, pp. 58–67.

[12] J. Current and M. Marish (1993), Multiobjective Transportation network design and routing problems, *European Journal of Operations Research*, Vol. 65, pp. 4–19.

[13] J. A. Diaz (1978), Solving multiobjective transportation problem, *Ekonomicko-Matematicky Obzor*, Vol. 14, pp. 267–274.

[14] W. F. A. EI-Washed (2001), A multi-objective transportation problem under fuzziness, *Fuzzy Sets and Systems*, Vol. 117, pp. 27–33.

[15] E. L. Hannan (1981), Linear programming with multiple fuzzy goals, *Fuzzy Sets and Systems*, Vol. 6, pp. 235–248.

[16] M. L. Hussein (1998), Complete solutions of multiple objective transportation problems with possibilistic coefficients, *Fuzzy Sets and Systems*, Vol. 93, pp. 293–299.

[17] R. Isermann (1979), The enumeration of all efficient solutions for a linear multiple-objective transportation problems, *Naval Research Logistic Quarterly*, Vol. 26, pp. 123–139.

[18] H. L. Kuwano (1996), On the multi-objective linear programming problem: Goal programming approach, *Fuzzy Sets and Systems*, Vol. 82, pp. 57–64.

[19] H. Leberling (1981), On finding compromise solutions in multi criteria problems using the fuzzy min-operator, *Fuzzy Sets and Systems*, Vol. 6, pp. 105–118.

[20] S. M. Lee and L. J. Moore (1973), Optimizing transportation problems with multiple objectives, *AIEE Trans.*, Vol. 5, pp. 333–338.

[21] L. Li and K. K. Lai (2000), A fuzzy approach to the multiobjective transportation problem, Computers and *Operations Research*, Vol. 27, pp. 43–57.

[22] M. K. Luhandjula (1982), Compensatory operators in fuzzy programming with multiple objectives, Fuzzy *Sets and Systems*, Vol. 8, pp. 245–252.

[23] J. L. Ringeust and D. B. Rinks (1987), Interactive solutions for the linear multiobjective transportation problem, *European Journal of Operational Research*, Vol. 32, pp. 96–106.

[24] M. Sakawa (1988), An interactive fuzzy satisficing method for multiobjective linear fractional programming problems, *Fuzzy Sets and Systems*, Vol. 28, pp. 129–144.

[25] G. L. Thompson (1997), Network models, in *Advances in sensitivity analysis and parametric programming*, T. Gal. and H. J. Greenberg (eds.), Kluwer, Boston, pp. 7–1–7–34.

[26] R. C.Wang and T. F. Liang (2004), Application of fuzzy multi-objective linear programming to aggregate production planning, *Computers and Industrial Engineering*, Vol. 46, pp. 17–41.

[27] H. J. Zimmermann (1976), Description and optimization of fuzzy systems, *International Journal of General Systems*, Vol. 2, pp. 209–215.

[28] H. J. Zimmermann (1978), Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems*, Vol. 1, pp. 45–56.

[29] H. J. Zimmermann (1996), *Fuzzy set theory and its application*, Kluwer, Boston.

[30] H. J. Zimmermann (1997), Fuzzy linear programming, in *Advances in sensitivity analysis and parametric programming*, T. Gal and H. I. Greenberg (eds.), Kluwer, Boston, pp. 15–1–15–40.

Received April, 2005