

Optimal lot sizing in manufacturing revisited

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Abstract

The lot sizing problem with non-instantaneous delivery and continuous consumption of manufactured items has been extensively investigated incorporating various non-manufacturing related cost and lot size variables. When the production cost per unit and the time per unit are functions of controllable manufacturing process variables, the selection of process variables impacts on the total stocking cost and the Economic Lot Size (EOQ). The EOQ in this context does not necessarily correspond to what may result if minimum production cost parameters are used. Furthermore, other constraints like maximum in-process inventory allowed, range of production rate allowed etc requires non-linear optimization techniques to solve the EOQ problem. This paper discusses these issues.

Keywords : *Inventory control, optimization, machining economics.*

Introduction

The traditional approach to finding the optimal lot size is based on balancing the annual carrying cost and annual setup cost. When quantity discounts are present, then the acquisition cost of the item is also taken into account in determining the economic order quantity when purchasing or the economic lot size when manufacturing. A comprehensive survey of research in this area has been given by Elmaghraby [4]. Karmakar [5] has proposed a solution to the batching problem by incorporating lead time in lot sizing. Ramaswamy and Lambert [7] treated processing cost

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and time per unit as functions of process variables in machining context to optimize total cost comprising machining cost, in-process inventory cost and due date violation cost. Koulmas [6] has developed queuing model with service with the service station subject to breakdown and determined analytically the optimal batch size for a machined product. Robinson Jr. and Sahin [10] consider a variation of the lot sizing problem by taking into consideration additional fixed charges, such as cleanup or inspection costs, associated with each time period's production. In many manufacturing situations, such as machining, the processing time and related cost per unit of product are functions of the processing variables which are controllable. When this is the case, the problem of determining the economic lot size is more complex as it becomes a function of processing variables also. Ramaswamy [8] investigated the selection of machining variables to optimize total cost for job lot production with a machining center incorporating machining cost and in-process inventory carrying cost. Ramaswamy [9] recently addressed the optimal lot sizing problem where the processing cost and time per unit of product were functions of the processing variables. The purpose of this presentation was to illustrate through rather simplified models the impact of time/cost variability, in a production context, on the economic lot size determination. This presentation will be reviewed for expositional convenience first and, later, an economic lot sizing problem in the context of machining economics will be presented.

We first consider a quadratic model where the processing cost per unit of product is a quadratic function of the processing time per unit.

Problem formulation

Notations

Let

- D = uniform constant annual demand for a manufactured item,
- S = tooling cost/setup,
- c = carrying cost per unit per year,
- d = demand rate expressed in units consistent with production rate,
- p = production rate,
- C = production cost per unit.

Now for any lot size, Q , the annual stocking cost is given by

$$\begin{aligned} TSC &= \text{carrying cost} + \text{setup cost} + \text{acquisition cost} \\ &= (p - d)Qc/(2p) + DS/Q + DC. \end{aligned} \quad (1)$$

When there is no quantity discounts, the acquisition cost is not relevant and the economic lot size minimizing TSC is given by the well known equation,

$$Q^* = \text{SQRT}[2DSp/(c(p - d))]. \quad (2)$$

Consider now the case where the production cost per unit, C , is a function of production time, t , per unit and is given by

$$C = a - bt + kt^2 \quad (3)$$

where a, b , and k are constants applicable to the specific manufacturing process.

A quadratic form for the function is assumed.

By solving $dc/dt = 0$ it can be shown that at minimum production cost, C_{\min} , the corresponding processing time, t_{\min} , are given by

$$t_{\min} = b/2k. \quad (4)$$

The minimum production cost is given by

$$C_{\min} = a - b^2/(4k). \quad (5)$$

Since a finite amount of time will always be required to produce a unit of product, a lower limit, t_1 , is imposed on the processing time, t . Also we impose an upper limit t_u , again, as a production rate constraint.

Now the average stocking cost per unit time, consistent with units for d and p , is given by

$$TSC = (p - d)Qc/(2p) + dS/Q + d(a - bt + kt^2). \quad (6)$$

Note. Units for c will be consistent with those of p and d .

Since $p = 1/t$, substituting for p , we can now state the problem as:

$$\text{Minimize } TSC = (1 - dt)tQc/2 + dS/Q + d(a - bt + kt^2) \quad (7)$$

$$\text{Subject to } t \geq t_1 \quad (8)$$

$$t \leq t_u \quad (9)$$

$$p \geq d \text{ or, equivalently, } 1/t \geq d \quad (10)$$

$$I_{\max} \leq M \quad (11)$$

Where

$$I = (p - d)Q/p, \text{ is the maximum built up in-process inventory (12)}$$

$$M = \text{Maximum in-process inventory allowed based on space} \\ \text{or management specified constraint (13)}$$

The above is a non-linear programming problem and can be solved using the EXCEL SOLVER ADD-IN Optimizing Software that comes with Windows 2000 or earlier versions. Table 1, below presents an example.

Table 1

Example

Problem data

d	c	S	a	B	k	t_l	t_u
2	0.01	500	5	2	10	0.03	0.3

Solution based on t_{cmin} and C_{min}

t_{cmin}	p_{cmin}	Q_{cmin}	C_{min}	TSC_{cmin}
0.1	10	500	4.90	13.80

Optimal solution

t^*	p^*	Q^*	C^*	TSC^*
0.262	3.816	648.308	5.163	13.410

Units

d : units/hour

c : \$ per unit per hour

S : \$ per setup

t : hours

p : units/hour

C : \$ per unit

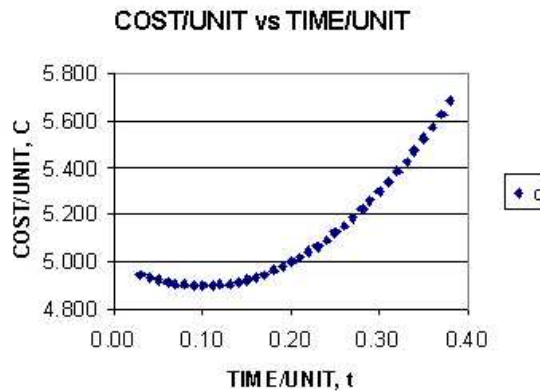
Q : units/lot

TSC : \$/hour

a, b, k : constants associated with the cost function, $C = a - bt - kt^2$

Optimal solution obtained using EXCEL SOLVER

A graph of C versus t is presented below.



The economic lot size is different from what would be obtained if processing time (t_{\min}) giving minimum production cost (C_{\min}) is used as illustrated in the above example. Another interesting result noticed in the above example is that the optimal processing time (t^*) in the optimal solution is greater than the processing time (t_{\min}). Because of the quadratic nature of the cost function, normally, the region where $t > t_{\min}$ will not be of interest as, in this region, both t and C will continuously increase. In the next section an application problem in the area of machining economics is presented.

Application example

Now consider the following example in machining economics where the processing time and corresponding cost per unit of product are functions of the processing variables cutting speed and tool feed rate. The cost, C , and the processing time, t , for machining a component involving a one pass operation can be expressed as,

$$t = t_L + t_c + t_d(t_c/T) \quad (14)$$

$$C = tX + Y(t_c/T) \quad (15)$$

where

t_L = loading and unloading time per component in minutes,

$t_c = Q/(Vf)$, cutting time in minutes per component,

- Q = some constant depending on the configuration of the machined part (for example in a single pass turning operation, $Q = \pi D l / 12$ where D = diameter of the work piece in inches, and l = length of the work piece in inches),
 v = cutting speed at the tool point in surface feet per minute,
 f = tool feed rate in inch/rev. or equivalent,
 T = $K/(v^{1/n} f^{1/n_1})$, Taylor's [9] tool life equation giving the tool life in minutes for the given tool-work combination and machining parameters, v , f , and depth of cut.
 K = constant in the tool life equation,
 $1/n$ = exponent of cutting speed in the tool life equation,
 $1/n_1$ = exponent of feed rate in the tool life equation,
 X = cost per minute of operator and machine in \$/min,
 Y = cost of new cutting edge or tool in \$/edge, and
 t_d = time for replacing a worn out edge or tool in min.

Substituting for t, t_c, T in terms V and f , the minimum C or t for a given feed, f , are obtained by solving respectively $\partial C/\partial V = 0$ and $\partial t/\partial V = 0$ and then substituting the corresponding optimal V and T in equations 15 and 14 respectively. Thus, for a given feed, f , the minimum cost tool life T_{cmin} and the maximum production rate tool life T_{tmin} are given by

$$T_{cmin} = (1/n - 1)(t_d X + Y)/X \quad (16)$$

$$T_{tmin} = (1/n - 1)t_d. \quad (17)$$

The corresponding cutting speed, V_c and V_t can be obtained through the tool life equation. It has been shown that for a given tool feed, both C and t are unimodal smooth convex functions of V , the cutting speed (see Armarego and Brown [1], and Brewer [2]). Brown [3] has shown that the best results are obtained by setting the feed, f , at as high a value as feasible (e.g. surface finish constraint) and then use the corresponding values of V_c and V_t to obtain the optimal machining cost and maximum production rate. For expositional convenience we consider the production of a cylindrical shaft with a constant uniform demand. Table 2 gives the problem data and solutions for TSC corresponding to minimum production cost, maximum production rate and the optimal solution for an example. Again, EXCEL SOLVER was used to solve the problem.

Table 2

Problem data: $D = 1.5''$, $L = 12''$, $n = 0.02$, $n_1 = 0.04$,
 $K = 5.5(10)^5$, $t_d = 1.5$ minutes per change, $V_{\max} = 200'$ /minute,
 $X = \$0.30$ /minute, $Y = \$5$ /cutting edge, $d = 0.4$ Units/Min.,
demand rate, $t_L = 0.5$ minutes

Note. Depth of cut is assumed to be fixed

Lot sizing example in machining environment

D	L	n	n_1	K	f
0.5	4	0.2	0.4	0.50E+05	0.005

X	Y	t_d	S	c	d
0.3	3	1.5	200	0.001	0.4

t_L	V_{\max}	T_{cmin}	T_{tmin}
0.5	200	46	6

V_{cmin}	V_{tmin}
100.89	151.62

t_{cmin}	t_{min}
1.57722	1.3636

p_{cmin}	p_{min}
0.6360	0.5473

C_{cmin}	C_{tmin}
0.5394	0.5473

EOQ_{cmin}	EOQ_{tmin}
774.98	593.30

TSC_{cmin}	TSC_{tmin}
0.4628	0.4886

Optimal solution		
V^*	C^*	
70.21	894.42	
t^*	p^*	T^*
2.0000	0.5000	281.80
C^*		
0.6159		
TSC^*		
0.4252		

Note. TSC is the average stocking cost per minute.

$$t^* > t_{\text{cmin}} > t_{\text{min}} \text{ and } TSC^* < TSC_{\text{cmin}} < TSC_{\text{tmin}}$$

Conclusions

In both the models considered with non-linear time cost relationships, the production cost per unit did not correspond to the minimum production cost per unit. This situation can be explained using the following reasoning: If production cost was not a factor in lot sizing, then the best policy would be to set the production rate equal to the demand rate and eliminate inventory carrying cost. The only cost incurred will be one setup cost during the product's life. However, when production cost as a function of production time is present, slowing the production rate to approach demand rate may result in excessive production cost which will offset the benefit of decreased inventory carrying cost. It is also possible that a production time/unit greater than that for minimum production cost could be optimal because of carrying cost reduction per setup. The normal tendency to ignore the region, where both production time per unit and the production cost per unit increase, in deciding process variable settings could lead to sub-optimal solutions in the context of total stocking cost minimization. In the machining environment problem, the production time per unit was larger than that at the minimum machining cost. Normally only the region from t_{cmin} (minimum cost production time) to t_{tmin} (minimum production time per unit) would be considered in selecting machining parameters. This paper clearly points to the need to look at production parameters when the production cost and time per unit of a product are functions thereof when optimizing total stocking cost.

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