

Fuzzy subjective game considering non-additive feelings of players

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Abstract

In the field of non-cooperative game Nakai [3] proposes a subjective game to explain the variety in a strategy selection by a player. But in constructing a subjective game by a motive distribution, the expected payoff is calculated additively. In actuality the degree of satisfaction for a player is generally very subjective and non-additive. Therefore this paper proposes a fuzzy subjective game in which we represent the non-additive feelings of a player by a fuzzy measure and evaluate the expected payoff by the fuzzy integral.

Keywords : *Non-cooperative game, non-additive feelings, fuzzy measure, fuzzy integral, subjective game.*

1. Introduction

In the field of non-cooperative game Nash [4] proposes a concept of equilibrium which is called the Nash equilibrium today, which is used widely and deeply as an important analysis instrument in the micro-economics. But up to now it is indicated that the Nash equilibrium point (NEP) has the following two problems:

- (1) There are many non-cooperative games with plural NEPs. In these case the NEP is not a suitable guide for strategy selection by a player.

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- (2) In many actual game experiments it is reported that even if the game has a unique NEP, many players do not follow their Nash equilibrium strategies (NES). For example, Minas et al. [2] has reported that in a prisoner's dilemma game thirty-six per cent of subjects select a non-NES "Cooperation".

Then it is hoped that the construction of a model can diminish the number of NEPs and explain the variety in strategy selection by a player. Nakai [3], Furuyama and Nakai [1] propose a subjective game for the purpose of dissolving these problems. In a subjective game we consider some motives for selecting a strategy and a subjective motive distribution of each player which a certain player P thinks subjectively. We define the subjective game for player P and consider that player P follows its NEP. This method can solve the above-mentioned problem (1) partially and problem (2) fundamentally. But in a subjective game, when it is constructed from a motive distribution, the expected payoffs are calculated additively. That is to say, if the payoff by the achievement of the motive m_1 (m_2) is a (b), the payoff by the achievement of both motives is represented $a + b$. Then the expected payoff under a motive distribution $\langle \lambda, 1 - \lambda \rangle$ is given by $\lambda a + (1 - \lambda)b$. But in actuality the degree of satisfaction is generally non-additive. For example, the payoff in the case that two motives m_1 and m_2 are achieved at the same time is larger than $a + b$, the sum of two payoffs by the achievement of each motive. In this paper in order to consider the non-additive feelings of players we utilize a fuzzy measure (λ -fuzzy measure). Furthermore we use an evaluated value by a fuzzy integral (the Choquet integral) as the expected payoff constructing the subjective game. This subjective game considering non-additive feelings with respect to motives is called a fuzzy subjective game.

2. Fuzzy subjective game

We define a n -person, non-cooperative finite game G as follows:

- $S_i = \{\alpha_{i1}, \dots, \alpha_{im_i}\}$: the set of pure strategies for player i ($= 1, \dots, n$).
The integer m_i denotes the number of possible pure strategies for player i .
- $a_i(\alpha_1, \dots, \alpha_n)$: the payoff of player i when player $1, \dots, n$ use pure strategies $\alpha_1, \dots, \alpha_n$ respectively ($\alpha_j \in S_j$, $j = 1, \dots, n$).

$x_i = \langle x_{i1}, \dots, x_{im_i} \rangle$: a mixed strategy for player i where x_{ij} is a probability that player i uses a pure strategy α_{ij}
 $a_i(x_1, \dots, x_n)$: the expected payoff of player i when player $1, \dots, n$ use mixed strategies x_1, \dots, x_n respectively.

Then

$$a_i(x_1, \dots, x_n) = \sum_{j_1=1}^{m_1} \cdots \sum_{j_n=1}^{m_n} a_i(\alpha_{1j_1}, \dots, \alpha_{nj_n}) x_{1j_1} \cdots x_{nj_n}. \quad (1)$$

Next we construct a fuzzy subjective game for a certain player P (one of players $1, \dots, n$). We consider l motives m_1, \dots, m_l , for selecting a strategy by a player. It is assumed that player P thinks that player i follows the motive selected by his subjective motive distribution $\theta^i = \langle \theta_1^i, \dots, \theta_l^i \rangle$ where θ_k^i is a probability that player P thinks that player i follows motive m_k . Of course $0 \leq \theta_k^i \leq 1$, $\sum_{k=1}^l \theta_k^i = 1$.

Definition 1. A set function $g(\cdot)$ defined on the family of subsets of the motive set $M = \{m_1, \dots, m_l\}$ is a fuzzy measure if and only if it satisfies the following three relations:

- (1) $g(\phi) = 0$
- (2) $g(M) = 1$
- (3) $A \subseteq B (\subseteq M) \Rightarrow g(A) \leq g(B)$ (simplicity) (2)

Definition 2. For a constant $\lambda (> -1)$, a fuzzy measure $g_\lambda(\cdot)$ having the following property is called λ -fuzzy measure: For any $A, B (\subseteq M)$, if $A \cap B = \phi$, then

$$g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B). \quad (3)$$

Remark 1. The λ -fuzzy measure $g_\lambda(A)$ denotes the degree of satisfaction when all motives belonged to the set $A (\subseteq M)$ are achieved. λ is a parameter denoting the degree of interaction. When $\lambda = 0$, λ -fuzzy measure $g_\lambda(\cdot)$ has the additivity. If $\lambda > (<) 0$, it has the multiplier (offset) effect.

For a motive distribution θ^i of player i we consider a λ -fuzzy measure satisfying the relation

$$g_\lambda^i(\{m_k\}) = \beta \theta_k^i \quad (k = 1, \dots, l) \quad (4)$$

where β is a non-negative constant. Then by the relation (3), for any subset $\{m_{k(1)}, \dots, m_{k(q)}\}$ of the set M we obtain

$$g_\lambda^i(\{m_{k(1)}, \dots, m_{k(q)}\}) = \sum_{j=1}^q \lambda^{j-1} \beta^j \sum_{(p_1, \dots, p_j)} \theta_{p_1}^i \theta_{p_2}^i \cdots \theta_{p_j}^i \quad (5)$$

where $\sum_{(p_1, \dots, p_j)}$ denotes the sum of all cases in sampling j elements from the set $\{k(1), \dots, k(q)\}$. Furthermore from $g_\lambda^i(M) = 1$, we obtain

$$g_\lambda^i(M) = \sum_{j=1}^l \lambda^{j-1} \beta^j \sum_{(p_1, \dots, p_j)} \theta_{p_1}^i \theta_{p_2}^i \cdots \theta_{p_j}^i = 1. \quad (6)$$

Namely β is the non-negative solution of the l -dimensional equation (6). Substituting the value of β into the equation (5), we can obtain the λ -fuzzy measure $g_\lambda^i(\cdot)$.

Next we evaluate the payoff of player i by the Choquet integral based on the λ -fuzzy measure $g_\lambda^i(\cdot)$. Let $a_i^k(\alpha_1, \dots, \alpha_n)$ be the payoff of player i in the case that player i follows the motive m_k and that player $1, \dots, n$ use strategies $\alpha_1, \dots, \alpha_n$ respectively. The payoff $a_i^k(\alpha_1, \dots, \alpha_n)$ is formed by the original games $a_i(\alpha_1, \dots, \alpha_n)$ ($i = 1, \dots, n$). For example, if the motive of player i is selfish, then

$$a_i^k(\alpha_1, \dots, \alpha_n) = a_i(\alpha_1, \dots, \alpha_n). \quad (7)$$

If the motive of player i is cooperative, then

$$a_i^k(\alpha_1, \dots, \alpha_n) = \frac{1}{n} \sum_{j=1}^n a_j(\alpha_1, \dots, \alpha_n). \quad (8)$$

Definition 3. Let $g(\cdot)$ be a fuzzy measure defined on the family of subsets of the set M . Suppose that a function $h(\cdot)$ defined on the set M satisfies the following relation (if necessary, change the number of each motive)

$$h(m_1) \geq h(m_2) \geq \cdots \geq h(m_l). \quad (9)$$

The Choquet integral of the function $h(\cdot)$ by the fuzzy measure $g(\cdot)$ is defined by

$$(C) \int h dg \equiv \sum_{k=1}^l [h(m_k) - h(m_{k+1})] g(M_k) \quad (10)$$

where $M(k) = \{m_1, \dots, m_k\}$ and $h(m_{l+1}) = 0$.

Letting t be the number of different values of l payoffs $a_i^k(\alpha_1, \dots, \alpha_n)$ ($k = 1, \dots, l$), we rearrange these payoffs in order of size from largest to smallest, that is,

$$\begin{aligned} a_i^1(\alpha_1, \dots, \alpha_n) &= \dots = a_i^{k_1}(\alpha_1, \dots, \alpha_n) > a_i^{k_1+1}(\alpha_1, \dots, \alpha_n) \\ &= \dots = a_i^{k_2}(\alpha_1, \dots, \alpha_n) > a_i^{k_2+1}(\alpha_1, \dots, \alpha_n) = \dots = a_i^{k_3}(\alpha_1, \dots, \alpha_n) \\ &> \dots > a_i^{k_{t-1}+1}(\alpha_1, \dots, \alpha_n) = \dots = a_i^{k_t}(\alpha_1, \dots, \alpha_n) \end{aligned} \quad (11)$$

where $k_{z-1} + 1 \leq k_z$ ($z = 1, \dots, t$) and $k_0 = 0, k_t = l$.

Then by the Choquet integral the evaluated value $\tilde{a}_i(\alpha_1, \dots, \alpha_n)$ of the payoff of player i is represented as follows:

$$\tilde{a}_i(\alpha_1, \dots, \alpha_n) = \sum_{z=1}^t \{a_i^{k_z}(\alpha_1, \dots, \alpha_n) - a_i^{k_z+1}(\alpha_1, \dots, \alpha_n)\} g_\lambda(M_{k_z}) \quad (12)$$

where $a_i^{k_t+1}(\alpha_1, \dots, \alpha_n) = a_i^{l+1}(\alpha_1, \dots, \alpha_n) = 0$.

Remark 2. If $\lambda = 0$ (additive case), then $\beta = 1$ by the equation (6). Therefore from (5) and (12) we obtain

$$g_0^i(\{m_{k(1)}, \dots, m_{k(q)}\}) = \sum_{i=1}^q \theta_{k(i)}^i \quad (13)$$

$$\tilde{a}_i(\alpha_1, \dots, \alpha_n) = \sum_{k=1}^l a_i^k(\alpha_1, \dots, \alpha_n) \theta_k^i \quad (14)$$

which coincides with the case of the traditional subjective game.

We define a fuzzy subjective game for player P by

$$\tilde{G}_P = [\tilde{a}_i(\alpha_1, \dots, \alpha_n) \mid i = 1, \dots, n; \alpha_j \in S_j (j = 1, 2, \dots, n)] \quad (15)$$

which reflects non-additive feelings of player P .

Definition 4. The n -tuple $x^* = (x_1^*, \dots, x_n^*)$ is the fuzzy subjective NEP if and only if x^* is the NEP of the fuzzy subjective game \tilde{G}_P , that is,

$$\tilde{a}_i(x_1^*, \dots, x_n^*) = \max_{x_i} \tilde{a}_i(x_1^*, \dots, x_{i-1}^*, x_i, x_{i+1}^*, \dots, x_n^*) \quad (i = 1, \dots, n). \quad (16)$$

Therefore it is desirable that player P uses the strategy indicated by his fuzzy subjective NEP.

3. Example

We consider a chicken game in which two players I, II select one of two actions A (avoid) and B (break) simultaneously. The payoff bimatrix is given by Figure 1.

		II	
		A (avoid)	B (break)
$G : I$	A (avoid)	0, 0	-1, 2
	B (break)	2, -1	-4, -4

Figure 1
The original game (chicken game)

Case 1. The case of the ordinary non-cooperative game.

When we don't consider subjective feelings of players, three ordinary NEPs exist. We list them with the corresponding payoffs.

$$\text{NEP (1): } (\langle 1, 0 \rangle, \langle 0, 1 \rangle) \quad \text{payoff: } (-1, 2)$$

$$\text{NEP (2): } (\langle 0, 1 \rangle, \langle 1, 0 \rangle) \quad \text{payoff: } (2, -1)$$

$$\text{NEP (3): } \left(\left\langle \frac{3}{5}, \frac{2}{5} \right\rangle, \left\langle \frac{3}{5}, \frac{2}{5} \right\rangle \right) \quad \text{payoff: } \left(-\frac{2}{5}, -\frac{2}{5} \right)$$

For example, at NEP(3), player I uses a pure strategy A(B) with probability $\frac{3}{5}$ ($\frac{2}{5}$) and player II uses also the same mixed strategy. Both players obtain the same expected payoff $\left(-\frac{2}{5} \right)$. In this example, since there are plural NEPs, a player can't decide which motive he should follow.

In the following two cases we consider the subjective game for player P (player I or II). Suppose that player P considers three motives in selecting a strategy.

m_1 : a selfish motive (maximization of his own payoff)

m_2 : an offensive motive (minimization of the opponent's payoff)

m_3 : a self-preservative motive (maximization of his own minimum payoff)

Let G_{ij} be a simple subjective game in which player I, II follow motives m_i, m_j respectively. The payoff of each simple subjective game is given as follows:

$G_{11} =$	$\begin{matrix} 0, 0 & -1, 2 \\ 2, -1 & -4, -4 \end{matrix}$	$G_{12} =$	$\begin{matrix} 0, 0 & -1, 1 \\ 2, -2 & -4, 4 \end{matrix}$	$G_{13} =$	$\begin{matrix} 0, -1 & -1, -4 \\ 2, -1 & -4, -4 \end{matrix}$
$G_{21} =$	$\begin{matrix} 0, 0 & -2, 2 \\ 1, -1 & 4, -4 \end{matrix}$	$G_{22} =$	$\begin{matrix} 0, 0 & -2, 1 \\ 1, -2 & 4, 4 \end{matrix}$	$G_{23} =$	$\begin{matrix} 0, -1 & -2, -4 \\ 1, -1 & 4, -4 \end{matrix}$
$G_{31} =$	$\begin{matrix} -1, 0 & -1, 2 \\ -4, -1 & -4, -4 \end{matrix}$	$G_{32} =$	$\begin{matrix} -1, 0 & -1, 1 \\ -4, -2 & -4, 4 \end{matrix}$	$G_{33} =$	$\begin{matrix} -1, -1 & -1, -4 \\ -4, -1 & -4, -4 \end{matrix}$

Figure 2
Simple subjective games

Suppose that player P considers that player I, II follow motive distributions $\langle \frac{2}{3}, \frac{1}{3}, 0 \rangle, \langle \frac{2}{3}, 0, \frac{1}{3} \rangle$, respectively. Namely player I gives priority to the maximization of his own payoff and hopes slightly the minimization of the opponent's payoff, but has no care about the worst case. On the other hand player II gives priority to the maximization of his own payoff and worries slightly about the worst case, but has no care about the minimization of the opponent's payoff.

Case 2. The case of the ordinary subjective game.

We consider the additive subjective feelings of players. The subjective game for player P is given by

$$G_P = \frac{2}{3} \cdot \frac{2}{3} G_{11} + \frac{2}{3} \cdot \frac{1}{3} G_{13} + \frac{1}{3} \cdot \frac{2}{3} G_{21} + \frac{1}{3} \cdot \frac{1}{3} G_{23} \tag{17}$$

which is written concretely in Figure 3.

		II	
		A	B
$G_P : I$	A	$0, -\frac{1}{3}$	$-\frac{4}{3}, 0$
	B	$\frac{5}{3}, -1$	$-\frac{4}{3}, -4$

Figure 3
The ordinary subjective game for player P

The subjective game G_P has the following NEPs (namely, the subjective NEPs).

$$\text{NEP(4): } (\langle 0, 1 \rangle, \langle 1, 0 \rangle) \quad \text{payoff } \left(\frac{5}{3}, -1 \right)$$

$$\text{NEP(5): } (\langle x, 1-x \rangle, \langle 0, 1 \rangle) \quad \text{payoff } \left(-\frac{4}{3}, -4(1-x) \right)$$

where x is any real number such that $9/10 \leq x \leq 1$. In this case the expected payoff of each player represents not an objective value but a subjective value by player P . Therefore, for example, if player P (player I) aims to NEP(4) and selects the pure strategy B (break), then he hopes to obtain the payoff $5/3$, but his payoff is 2 actually. Even if we consider a subjective game, there are plural NEPs and they can't become a suitable guide for selecting a strategy.

Case 3. The case of a fuzzy subjective game.

We take into consideration non-additive feelings of players by a fuzzy measure and fuzzy integral.

We consider a λ -fuzzy measure satisfying the equation (4), that is,

$$g_\lambda(\{m_1\}) = \frac{2}{3}\beta, \quad g_\lambda(\{m_2\}) = \frac{1}{3}\beta, \quad g_\lambda(\{m_3\}) = 0 \quad (18)$$

where β is a non-negative constant. By the equation (3) we obtain

$$\begin{aligned} g_\lambda(\{m_1, m_3\}) &= \frac{2}{3}\beta, & g_\lambda(\{m_2, m_3\}) &= \frac{1}{3}\beta, \\ g_\lambda(\{m_1, m_2\}) &= g_\lambda(\{m_1, m_2, m_3\}) = \frac{2}{9}\lambda\beta^2 + \beta. \end{aligned} \quad (19)$$

From the condition $g_\lambda(\{m_1, m_2, m_3\}) = 1$, we can obtain

$$\beta = \begin{cases} (-9 + 3\sqrt{9 + 8\lambda})/(4\lambda) & \text{if } \lambda > 0 \\ 1 & \text{if } \lambda = 0 \\ (-9 - 3\sqrt{9 + 8\lambda})/(4\lambda) & \text{if } \lambda < 0. \end{cases} \quad (20)$$

Substituting (20) into (19), we can determine the λ -fuzzy measure $g_\lambda(\cdot)$ for a given constant $\lambda(> -1)$ completely.

Similarly for a given constant $\mu(> -1)$, we consider a μ -fuzzy measure satisfying the equation (4), that is,

$$g_\mu(\{m_1\}) = \frac{2}{3}\gamma, \quad g_\mu(\{m_2\}) = 0, \quad g_\mu(\{m_3\}) = \frac{1}{3}\gamma \quad (21)$$

where γ is a non-negative constant. By the equation (3) we obtain

$$\begin{aligned} g_\mu(\{m_1, m_2\}) &= \frac{2}{3}\gamma, & g_\mu(\{m_2, m_3\}) &= \frac{1}{3}\gamma, \\ g_\mu(\{m_1, m_3\}) &= g_\lambda(\{m_1, m_2, m_3\}) = \frac{2}{9}\mu\gamma^2 + \gamma. \end{aligned} \quad (22)$$

From the condition $g_\mu(\{m_1, m_2, m_3\}) = 1$, we can obtain

$$\gamma = \begin{cases} (-9 + 3\sqrt{9 + 8\mu})/(4\mu) & \text{if } \mu > 0 \\ 1 & \text{if } \mu = 0 \\ (-9 - 3\sqrt{9 + 8\mu})/(4\mu) & \text{if } \mu < 0. \end{cases} \quad (23)$$

Substituting (23) into (22), we can determine the μ -fuzzy measure $g_\mu(\cdot)$ for a given constant $\mu (> -1)$ completely.

Let $a_{ij}^k(b_{ij}^k)$ be the expected payoff of player I (II) when player I (II) follows the motive m_k and when player I, II uses strategies i, j (A or B) respectively. These values can be obtained from the original game G by the method shown in the previous section. We show these values in Table 1.

Table 1
The evaluation of players under each motive

	a_{AA}^k	a_{AB}^k	a_{BA}^k	a_{BB}^k	b_{AA}^k	b_{AB}^k	b_{BA}^k	b_{BB}^k
m_1	0	-1	2	-4	0	2	-1	-4
m_2	0	-2	1	4	0	1	-2	4
m_3	-1	-1	-4	-4	-1	-4	-1	-4

We evaluate these payoffs synthetically by using the Choquet integral based on the fuzzy measures $g_\lambda(\cdot)$ and $g_\mu(\cdot)$. Let $\tilde{a}_{ij}(\tilde{b}_{ij})$ be the expected payoff of player I (II) in the synthetic evaluation when player I, II uses strategies i, j (A or B) respectively. For example,

$$\tilde{a}_{AB} = -2 \times 1 + \{-1 - (-2)\} \times \frac{2}{3}\beta = \frac{2}{3}\beta - 2 \quad (24)$$

$$\tilde{b}_{AB} = -4 \times 1 + \{1 - (-4)\} \times \frac{2}{3}\gamma + \{2 - 1\} \times \frac{2}{3}\gamma = 4\gamma - 4. \quad (25)$$

Then we can obtain the fuzzy subjective game $\tilde{G}_P = [(\tilde{a}_{ij}, \tilde{b}_{ij}) \mid i, j = A, B]$ for player P which is shown in Figure 4.

		II	
		A	B
$\tilde{G}_P : I$	A	$0, \frac{2}{3}\gamma - 1$	$\frac{2}{3}\beta - 2, 4\gamma - 4$
	B	$\frac{2}{3}\beta + 1, -1$	$\frac{8}{3}\beta - 4, -4$

Figure 4
The fuzzy subjective game for player P

Let $\langle x, 1 - x \rangle$ ($\langle y, 1 - y \rangle$) be a mixed strategy of player I (II) where $x(y)$ is a probability that player I (II) uses a pure strategy A. Furthermore let $\langle \tilde{x}(y), 1 - \tilde{x}(y) \rangle$ be the optimal response strategy of player I given that player II uses a mixed strategy $\langle y, 1 - y \rangle$. Similarly let $\langle \tilde{y}(x), 1 - \tilde{y}(x) \rangle$ be the optimal response strategy of player II given that player I uses a mixed strategy $\langle x, 1 - x \rangle$. By the slightly complicated calculations, we can obtain the following results.

If $0 < \beta < 1$, then

$$\tilde{x}(y) = \begin{cases} 1 & \text{if } y < K \\ \text{any} & \text{if } y = K \\ 0 & \text{if } y > K \end{cases} \quad (26)$$

where $K = 6(1 - \beta)/(9 - 4\beta)$.

If $\beta = 1$, then

$$\tilde{x}(y) = \begin{cases} \text{any} & \text{if } y = 0 \\ 0 & \text{if } y > 0. \end{cases} \quad (27)$$

If $\beta > 1$, then $\tilde{x}(y) = 0$ for any y .

If $0 < \gamma < 9/10$, then $\tilde{y}(x) = 1$ for any x .

If $\gamma = 9/10$, then

$$\tilde{y}(x) = \begin{cases} 1 & \text{if } x < 1 \\ \text{any} & \text{if } x = 1. \end{cases} \quad (28)$$

If $\gamma > 9/10$, then

$$\tilde{y}(x) = \begin{cases} 1 & \text{if } x < 9/(10\gamma) \\ \text{any} & \text{if } x = 9/(10\gamma) \\ 0 & \text{if } x > 9/(10\gamma). \end{cases} \quad (29)$$

From the above-mentioned discussion, we can obtain the fuzzy subjective NEP for player P as follows:

- (i) If $-1 < \lambda < 0$ or $\mu > 5/9$, then there is a unique fuzzy subjective NEP $(\langle 0, 1 \rangle, \langle 1, 0 \rangle)$, that is, player I (II) uses the pure strategy B(A). In this case player I, II hope subjectively the expected payoff $\frac{2}{3}\beta + 1, -1$ respectively, but the actual payoffs are $2, -1$ respectively.
- (ii) If $\lambda > 0$ and $-1 < \mu < 5/9$, then there are three fuzzy subjective NEPs:

$$(\langle 0, 1 \rangle, \langle 1, 0 \rangle), (\langle 1, 0 \rangle, \langle 0, 1 \rangle) \\ (\langle 9/(10\gamma), 1 - 9/(10\gamma) \rangle, \langle K, 1 - K \rangle).$$

- (iii) If $\lambda = 0$ and $-1 < \mu < 5/9$, then there are innumerable (but two sorts) fuzzy subjective NEPs:

$$(\langle 0, 1 \rangle, \langle 1, 0 \rangle), (\langle x, 1 - x \rangle, \langle 0, 1 \rangle)$$

where x is any real number such that $9/(10\gamma) \leq x \leq 1$.

- (iv) If $\lambda > 0$ and $\mu = 5/9$, then there are innumerable (but two sorts) fuzzy subjective NEPs:

$$(\langle 0, 1 \rangle, \langle 1, 0 \rangle), (\langle 1, 0 \rangle, \langle y, 1 - y \rangle)$$

where y is any real number such that $0 \leq y \leq K$.

- (v) If $\lambda = 0$ and $\mu = 5/9$, then there are two fuzzy subjective NEPs:

$$(\langle 0, 1 \rangle, \langle 1, 0 \rangle), (\langle 1, 0 \rangle, \langle 0, 1 \rangle).$$

Note that considering the constants $\lambda > -1$ and $\mu > -1$, it can't occur that $3/2 \leq \beta \leq 3$ or $3/2 \leq \gamma \leq 3$.

We list some characteristics of the above fuzzy subjective NEPs.

- (1) If the motive evaluation of player I has the offset effect ($\lambda < 0$), or if the motive evaluation of player II has the multiplier effect strongly ($\mu > 5/9$), then the fuzzy subjective NEP is unique. It denotes that player I (II) uses the pure strategy "break" ("avoid").
- (2) If $\lambda \geq 0$ and $\mu \leq 5/9$ (that is, the motive evaluation of player I has the multiplier effect and that of player II has a little multiplier effect), then the fuzzy subjective NEP is not unique.

- (3) The pure NEP $(\langle 0, 1 \rangle, \langle 1, 0 \rangle)$ (that is, I uses “break” and II uses “avoid”) is always the fuzzy subjective NEP.
- (4) If $\lambda = \mu = 0$ (that is, both players have additive motive evaluations), then the result coincides with that in Case 2.

4. Discussion

In this paper we propose a fuzzy subjective game. A subjective game introduced by Nakai [3] intends to consider subjective feelings of players. But it is restricted to the case of additive evaluation of feelings. In actuality human feelings are non-additive in most cases and there are multiplier effects and offset effects. We represent the non-additivity of human feelings by a fuzzy measure, evaluate a payoff of a player by a fuzzy integral and construct his fuzzy subjective game. We consider that a player follows the strategy (the fuzzy subjective NES) indicated by the NEP of his own fuzzy subjective game. It is expected that the variety in selecting a strategy is explained by this idea. On the other hand it is a left problem to leave out impractical equilibria and to indicate a unique NEP as a suitable guide in all cases of strategy selections.

References

- [1] S. Furuyama and T. Nakai, The construction of subjective games by motive distributions in n -person non-cooperative game, *Journal of Information and Optimization Sciences*, Vol. 25 (3) (2004), pp. 533–541.
- [2] J. S. Minas, A. Scodel, D. Marlowe and H. Raweon, Some descriptive aspects of two-person non-zero-sum games II, *Journal of Conflict Resolution*, Vol. 4 (1960), pp. 193–197.
- [3] T. Nakai, Subjective games in a non-cooperative game, *Journal of Information and Optimization Sciences*, Vol. 21 (1) (2000), pp. 129–147.
- [4] J. F. Nash, Non-cooperative games, *Annals of Mathematics*, Vol. 54 (1951), pp. 286–295.

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