Some generalizations of Rédei's theorem

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Abstract

By the famous theorems of Rédei, a set of q points in $\mathrm{AG}(2,q)$ (respectively p points in $\mathrm{AG}(2,p)$, p prime) is either a line or it determines at least $\sqrt{q}+1$ (respectively $\frac{p+3}{2}$) directions. We generalize these results on two fronts. First we provide bounds on the number of directions determined by a set of $n \leq q$ points in a general projective plane of order q. Secondly, given a dual n-arc in $\Pi = \mathrm{PG}(k,q)$ we consider Π as embedded in $\Sigma = \mathrm{PG}(k+1,q)$ where $E = \Sigma - \Pi$ is the associated affine space. A collection of affine points is a *transversal set* of $\mathcal{K}$ if any line incident with a k-fold point of $\mathcal{K}$ is incident with at most one point of S. We reformulate Rédei's results in the plane as results on transversal sets. In this setting we generalize Rédei's theorems to higher dimensions. We also provide a new proof of a well known theorem on extending arcs in $\mathrm{PG}(k,q)$.

Keywords : *Arc, dual arc, Rédei's theorem.*

1. Introduction

In 1970, the results of Rédei [12] provided the following Theorem.

Theorem 1.1 (Rédei's Theorem). *Let $\pi = \mathrm{PG}(2,q)$ with a distinguished line ℓ_∞. Let $\mathcal{S}$ be a set of q points of $\pi - \ell_\infty$ and let $\mathcal{A}$ be a collection of δ points on ℓ_∞ with the following property. Any line through a point of $\mathcal{A}$ intersects $\mathcal{S}$ in at most one point. If*

$$\delta > \begin{cases} \dfrac{q-1}{2} & q \text{ is prime} \\ q - \sqrt{q} & \text{otherwise} \end{cases}$$

then $\mathcal{S}$ is a subset of a line of π.

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