On the extending of k-regular graphs and their strong defining spectrum

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Abstract

In a given graph $G = (V, E)$, a set of vertices S with an assignment of colours to them is said to be a defining set of the vertex colouring of G, if there exists a unique extension of the colours of S to a $c \geq \chi(G)$ colouring of the vertices of G. A defining set with minimum cardinality is called a minimum defining set and its cardinality is the defining number, denoted by $d(G, c)$. The defining set S is strong, if there exists an ordering $\{v_1, v_2, \ldots, v_{n-s}\}$ of the vertices of $\langle G - S \rangle$ such that in the induced list of colours in each of the subgraphs $\langle G - S \rangle$, $\langle G - S \cup \{v_1\} \rangle$, $\langle G - S \cup \{v_1, v_2\} \rangle$ and $\langle G - S \cup \{v_1, v_2, \ldots, v_{n-s}\} \rangle$, there exist at least one vertex whose list of colours is of cardinality 1. The strong defining number, $sd(G, c)$ of G is the cardinality of its smallest strong defining set.

If $\mathcal{F}$ is a family of graphs then $\mathrm{Spec}_c(\mathcal{F}) = \{d \mid \exists\, G, G \in \mathcal{F}, sd(G, c) = d\}$. Here we study the cases where $\mathcal{F}$ is the family of k-regular (connected and disconnected) graphs on n vertices and $c = k - 1$. Also the $\mathrm{Spec}_{k-1}(\mathcal{F})$ defining spectrum of all k-regular (connected and disconnected) graph on n vertices are verified for $k = 3$ and 4.

Keywords : *Regular graphs, colouring, defining spectrum.*

1. Introduction

A c-colouring (proper c-colouring) of a graph G is an assignment of c different colours to the vertices of G such that no two adjacent vertices receive the same colour. The vertex chromatic number of a graph G,

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