

The spectrum of nested group divisible designs of type t^n

Jinhua Wang*

School of Sciences

Nantong University

Nantong 226007

P. R. China

Abstract

The obvious necessary conditions for the existence of a λ -fold nested group divisible design of type t^n are $\lambda t(n-1) \equiv 0 \pmod{6}$ and $n \geq 4$. We show that these conditions are also sufficient.

Keywords : *Nested group divisible design, pairwise balanced design, skew room frame.*

1. Introduction

Let K be a set of positive integers, and let λ be positive integer. A *group divisible design* (GDD) with index λ is a triple $(X, \mathcal{G}, \mathcal{A})$ which satisfies the following

- (1) \mathcal{G} is a partition of X into subsets called *groups*.
- (2) \mathcal{A} is a set of subsets of X (called *blocks*) such that a group and a block contain at most one common point; and
- (3) every pair of points from distinct groups occurs in exactly λ blocks.

The *group type* (or type) of a GDD $(X, \mathcal{G}, \mathcal{A})$ is the multiset $\{|G| : G \in \mathcal{G}\}$. For convenience, we will use the “exponential” notation $g_1^{n_1} g_2^{n_2} \cdots g_t^{n_t}$ to denote a GDD with n_i groups of size g_i , $1 \leq i \leq t$ and $\sum_{i=1}^t n_i g_i = |X|$.

*E-mail: jhwang@ntu.edu.cn