

## Theory and design of $m$ -ary balanced codes which are invariant under symbol permutation

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### Abstract

A symbol permutation invariant balanced (SPI-balanced) code over the alphabet  $\mathbb{Z}_m = \{0, 1, \dots, m-1\}$  is a block code over  $\mathbb{Z}_m$  such that each alphabet symbol occurs as many times as any other symbol in every codeword. For this reason every permutation among the symbols of the alphabet changes a SPI-balanced code into a SPI-balanced code. This means that SPI-balanced words are “the most balanced” among all possible  $m$ -ary balanced word types, and this property makes them very attractive from the application perspective. In particular, they can be used to achieve  $m$ -ary DC-free communication, to detect/correct asymmetric/unidirectional errors on the  $m$ -ary asymmetric/unidirectional channel, to achieve delay-insensitive communication, to maintain data integrity in digital optical disks, and so on. The paper gives some efficient methods to convert (encode)  $m$ -ary information sequences into  $m$ -ary SPI-balanced codes whose redundancy is equal to roughly double the minimum possible redundancy  $r_{\min} \simeq [(m-1)/2] \log_m n - (1/2)[1 - (1/\log_{2\pi} m)]m - (1/\log_{2\pi} m)$  for SPI-balanced code with  $k$  information digits and length  $n = k + r$ . For example, the first method given in the paper encodes  $k$  information digits into a SPI-balanced code of length  $n = k + r$ , with  $r = (m-1) \log_m k + O(m \log_m \log_m k)$ . A second method is a recursive method, which uses the first as base code, and encodes  $k$  digits into a SPI-balanced code of length  $n = k + r$ , with  $r \simeq (m-1) \log_m n \log_m [(m-1)!]$ .

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**Keywords :** *Balanced codes,  $m$ -ary alphabet,  $m$ -th roots of unity, digital communication,  $m$ -ary communication, line codes, DC-free communication, asymmetric/unidirectional error detection/correction, delay-insensitive communication, data integrity.*

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